Iowa State University

ECpE Department

Modeling A Transformer in OpenDSS

Acknowledgement

[1] Kersting, William H. "Distribution system modeling and analysis." Electric power generation, transmission, and distribution. CRC press, 2018. 26-1.

[2] Dugan, Roger C., and T. McDermott. "Reference guide." The Open Distribution System Simulator (OpenDSS). EPRI (2016).

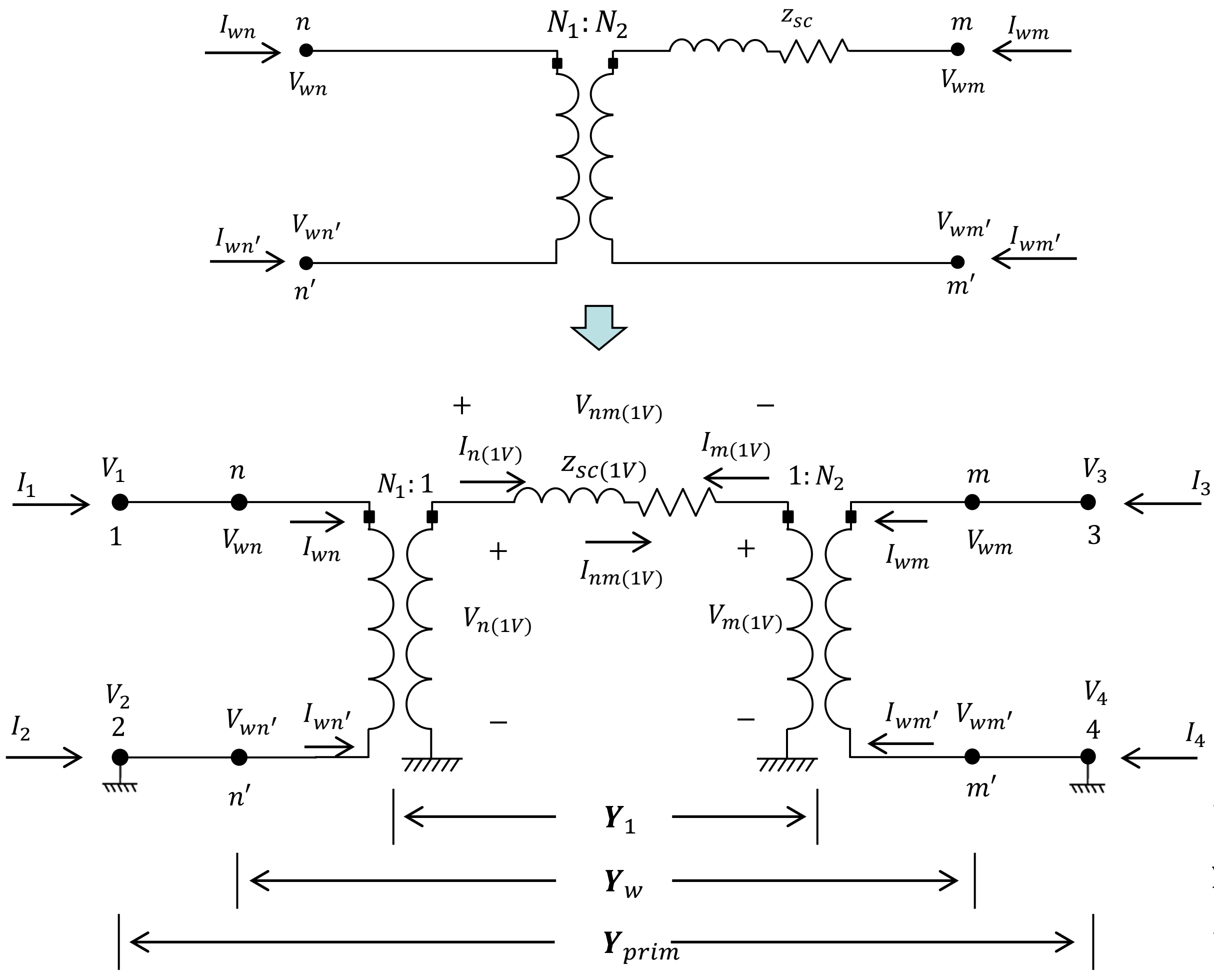
[3] "Transformer Element." https://youtu.be/z9EbQCmaWBo?si=4akygeT2wIpQ5l5U.

Modeling A Transformer in OpenDSS

1. Single-phase Transformer
2. Three-phase Delta-Grounded Wye Step-down Transformer
3. Three-phase Delta-Grounded Wye Step-up Transformer
4. Three-phase Ungrounded Wye-Delta Step-down Transformer
5. Three-phase Ungrounded Wye-Delta Step-up Transformer
6. Three-phase Grounded Wye-Delta Step-down Transformer
7. Three-phase Grounded Wye-Grounded Wye Transformer
8. Three-phase Delta-Delta Transformer

Modeling A Transformer in OpenDSS

1. Single-phase Transformer



$$\boldsymbol{Y}_1 = \boldsymbol{B} y_{sc(1V)} \boldsymbol{B}^{\mathrm{T}}$$

$$\boldsymbol{Y}_w = \boldsymbol{N} \boldsymbol{B} y_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{N}^T$$

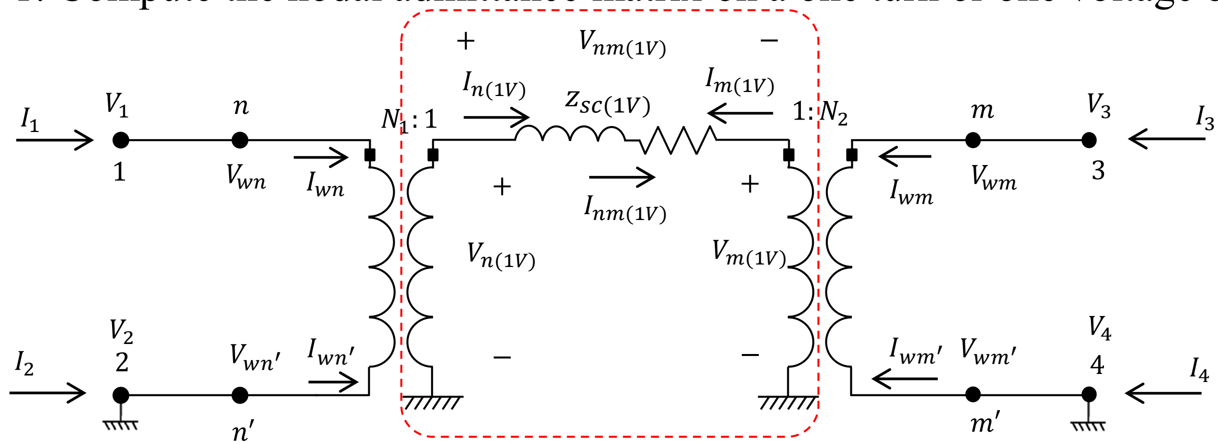
$$\boldsymbol{Y}_{prim} = \boldsymbol{A} \boldsymbol{N} \boldsymbol{B} y_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{N}^T \boldsymbol{A}^T$$

$z_{sc(1V)} = z_{sc}(pu) * z_{base(1V)}$, where, $z_{base(1V)} = \frac{1^2}{S_t} = \frac{1}{S_t}$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.



$$\begin{cases} I_{n(1V)} = I_{nm(1V)} \\ I_{m(1V)} = -I_{nm(1V)} \end{cases} \Rightarrow \begin{bmatrix} I_{n(1V)} \\ I_{m(1V)} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} I_{nm(1V)} \Rightarrow \boldsymbol{I}_1 = \boldsymbol{B} I_{nm(1V)}, \quad \text{where, } \boldsymbol{I}_1 = \begin{bmatrix} I_{n(1V)} \\ I_{m(1V)} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$$V_{nm(1V)} = V_{n(1V)} - V_{m(1V)} \Rightarrow V_{nm(1V)} = [1 \quad -1] \begin{bmatrix} V_{n(1V)} \\ V_{m(1V)} \end{bmatrix} \Rightarrow V_{nm(1V)} = \boldsymbol{B}^{\mathrm{T}} \boldsymbol{V}_1, \text{ where, } \boldsymbol{V}_1 = \begin{bmatrix} V_{n(1V)} \\ V_{m(1V)} \end{bmatrix}$$

Also, $V_{nm(1V)} = z_{sc(1V)} I_{nm(1V)}$, i.e., $I_{nm(1V)} = y_{sc(1V)} V_{nm(1V)}$

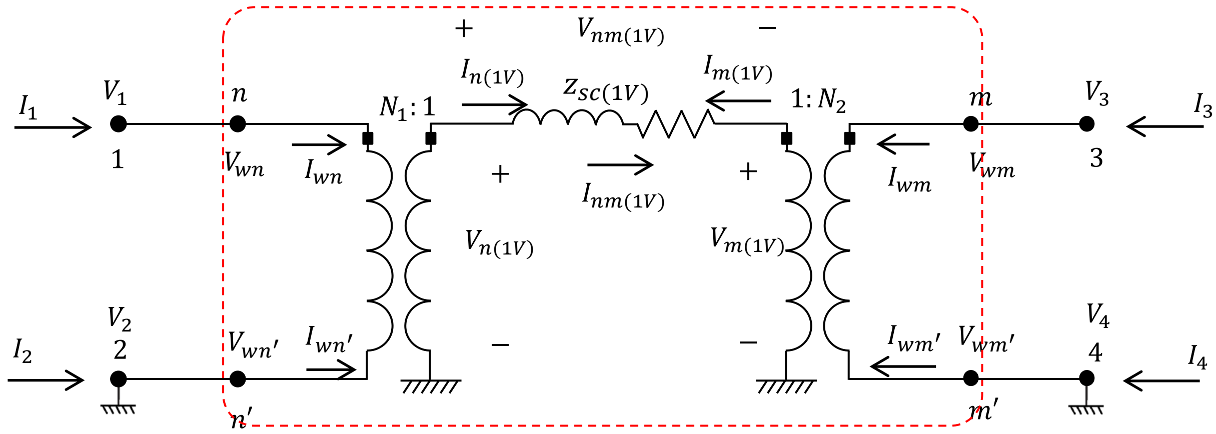
Let $\boldsymbol{I}_1 = \boldsymbol{Y}_1 \boldsymbol{V}_1$, where, $\boldsymbol{Y}_1$ is the admittance matrix on a one turn or one voltage base. What is $\boldsymbol{Y}_1$?

$$\left.\begin{matrix} \boldsymbol{I}_1 = \boldsymbol{Y}_1 \boldsymbol{V}_1 \\ \boldsymbol{I}_1 = \boldsymbol{B} I_{nm(1V)} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \boldsymbol{B} I_{nm(1V)} = \boldsymbol{Y}_1 \boldsymbol{V}_1 \\ V_{nm(1V)} = \boldsymbol{B}^{\mathrm{T}} \boldsymbol{V}_1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \boldsymbol{B} I_{nm(1V)} = \boldsymbol{Y}_1 (\boldsymbol{B}^{\mathrm{T}})^{-1} V_{nm(1V)} \\ I_{nm(1V)} = y_{sc(1V)} V_{nm(1V)} \end{matrix}\right\} \Rightarrow \boldsymbol{B} y_{sc(1V)} V_{nm(1V)} = \boldsymbol{Y}_1 (\boldsymbol{B}^{\mathrm{T}})^{-1} V_{nm(1V)} \Rightarrow \boldsymbol{Y}_1 = \boldsymbol{B} y_{sc(1V)} \boldsymbol{B}^{\mathrm{T}}$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 2: Consider winding turns ratio.



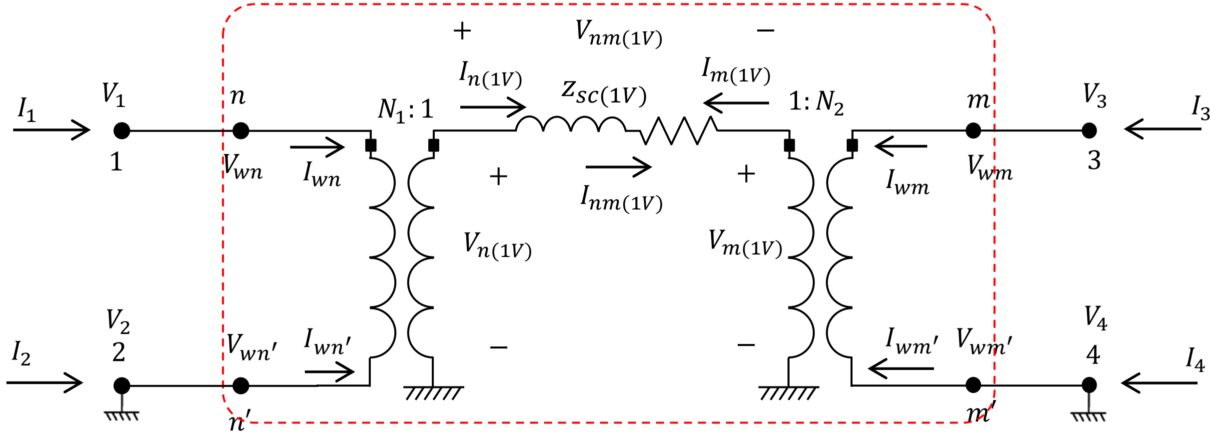
$$\begin{cases} I_{wn} = I_{n(1V)}/N_1 \\ I_{wn'} = -I_{n(1V)}/N_1 \\ I_{wm} = I_{m(1V)}/N_2 \\ I_{wm'} = -I_{m(1V)}/N_2 \end{cases} \Rightarrow \begin{bmatrix} I_{wn} \\ I_{wn'} \\ I_{wm} \\ I_{wm'} \end{bmatrix} = \begin{bmatrix} 1/N_1 & 0 \\ -1/N_1 & 0 \\ 0 & 1/N_2 \\ 0 & -1/N_2 \end{bmatrix} \begin{bmatrix} I_{n(1V)} \\ I_{m(1V)} \end{bmatrix} \Rightarrow \boldsymbol{I}_w = \boldsymbol{N}\boldsymbol{I}_1, \text{ where, } \boldsymbol{I}_w = \begin{bmatrix} I_{wn} \\ I_{wn'} \\ I_{wm} \\ I_{wm'} \end{bmatrix}, \ \boldsymbol{N} = \begin{bmatrix} 1/N_1 & 0 \\ -1/N_1 & 0 \\ 0 & 1/N_2 \\ 0 & -1/N_2 \end{bmatrix}$$

$$\begin{cases} V_{wn} - V_{wn'} = V_{n(1V)}N_1 \\ V_{wm} - V_{wm'} = V_{m(1V)}N_2 \end{cases} \Rightarrow \begin{bmatrix} V_{n(1V)} \\ V_{m(1V)} \end{bmatrix} = \begin{bmatrix} 1/N_1 & -1/N_1 & 0 & 0 \\ 0 & 0 & 1/N_2 & -1/N_2 \end{bmatrix} \begin{bmatrix} V_{wn} \\ V_{wn'} \\ V_{wm} \\ V_{wm'} \end{bmatrix} \Rightarrow \boldsymbol{V}_1 = \boldsymbol{N}^T\boldsymbol{V}_w, \text{ where,} \boldsymbol{V}_w = \begin{bmatrix} V_{wn} \\ V_{wn'} \\ V_{wm} \\ V_{wm'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 2: Consider winding turns ratio.



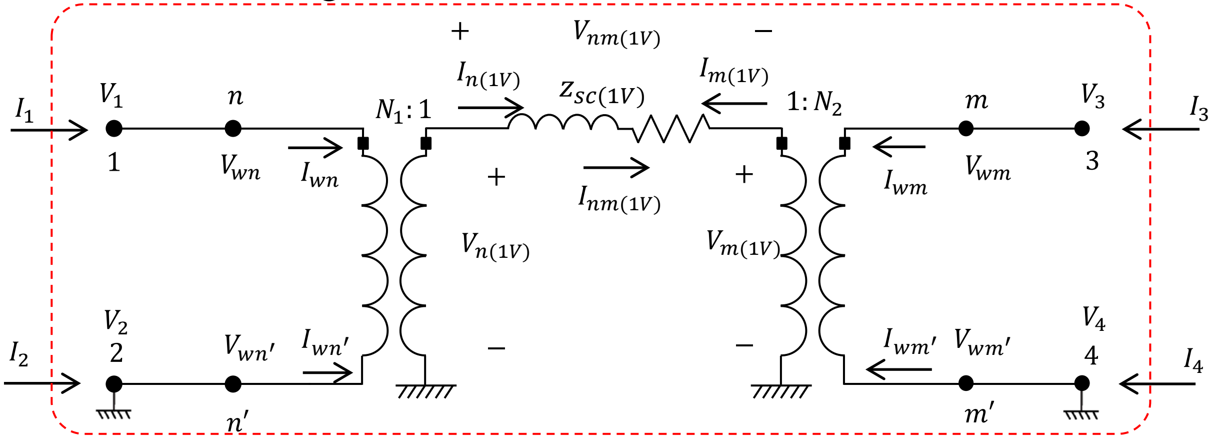
Next, let $\boldsymbol{I}_w = \boldsymbol{Y}_w \boldsymbol{V}_w$. How to obtain $\boldsymbol{Y}_w$?

$$\left.\begin{aligned}\boldsymbol{I}_w &= \boldsymbol{Y}_w \boldsymbol{V}_w\\ \boldsymbol{I}_w &= \boldsymbol{N}\boldsymbol{I}_1\\ \boldsymbol{V}_1 &= \boldsymbol{N}^T\boldsymbol{V}_w\end{aligned}\right\} \Rightarrow \left.\begin{aligned}\boldsymbol{N}\boldsymbol{I}_1 &= \boldsymbol{Y}_w(\boldsymbol{N}^T)^{-1}\boldsymbol{V}_1\\ \boldsymbol{I}_1 &= \boldsymbol{Y}_1\boldsymbol{V}_1\end{aligned}\right\} \Rightarrow \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{V}_1 = \boldsymbol{Y}_w(\boldsymbol{N}^T)^{-1}\boldsymbol{V}_1 \Rightarrow \left.\begin{aligned}\boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{N}^T\\ \boldsymbol{Y}_1 &= \boldsymbol{B}y_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}\end{aligned}\right\} \Rightarrow \boldsymbol{Y}_w = \boldsymbol{N}\boldsymbol{B}y_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

Step 3: Consider winding connections.



Let $\boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim}$. How to obtain $\boldsymbol{Y}_{prim}$?

$$\left.\begin{aligned} I_1 &= I_{wn} \\ I_2 &= I_{wn'} \\ I_3 &= I_{wm} \\ I_4 &= I_{wm'} \end{aligned}\right\} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{wn} \\ I_{wn'} \\ I_{wm} \\ I_{wm'} \end{bmatrix} \Rightarrow \boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_w, \text{ where, } \boldsymbol{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\left.\begin{aligned} V_1 &= V_{wn} \\ V_2 &= V_{wn'} \\ V_3 &= V_{wm} \\ V_4 &= V_{wm'} \end{aligned}\right\} \Rightarrow \begin{bmatrix} V_{wn} \\ V_{wn'} \\ V_{wm} \\ V_{wm'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \Rightarrow \boldsymbol{V}_w = \boldsymbol{A}^T\,\boldsymbol{V}_{prim}, \text{ where,} \boldsymbol{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}.$$

$$\left.\begin{aligned} \boldsymbol{I}_{prim} &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim} \\ \boldsymbol{I}_{prim} &= \boldsymbol{A}\boldsymbol{I}_w \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{I}_w &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim} \\ \boldsymbol{V}_w &= \boldsymbol{A}^T\,\boldsymbol{V}_{prim} \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \boldsymbol{I}_w &= \boldsymbol{Y}_w\,\boldsymbol{V}_w \\ \boldsymbol{A}\boldsymbol{I}_w &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^T)^{-1}\boldsymbol{V}_w \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{Y}_w\,\boldsymbol{V}_w &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^T)^{-1}\boldsymbol{V}_w \\ \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{B}y_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T \end{aligned}\right\} \Rightarrow \boldsymbol{Y}_{prim} = \boldsymbol{A}\boldsymbol{N}\boldsymbol{B}y_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T\,\boldsymbol{A}^T$$

Modeling A Transformer in OpenDSS

1. Single-phase Transformer

An example:

```
// Define a single-phase transformer.
New Transformer.xfmr phases=1 windings=2 xhl=5
~ wdg=1 bus=n  kV=66.39 kva=50 %r=2.5
~ wdg=2 bus=m  kV=4.16  kva=50 %r=2.5
```

- Transformer name
- Number of phases
- Number of windings
- Percent reactance high-to-low
- Specify which winding will be edited
- Rated voltage
- Base kVA rating
- Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{sc(1V)} = \frac{1}{z_{sc(1V)}} = \frac{1}{z_{sc}(pu) * z_{base(1V)}} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t}} = \frac{50000}{(0.05 + j0.05)} = 500000 - j500000\ \Omega$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} 1/66390 & 0 \\ -1/66390 & 0 \\ 0 & 1/4160 \\ 0 & -1/4160 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANB}y_{sc(1V)}\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\ \boldsymbol{A}^{T} =$

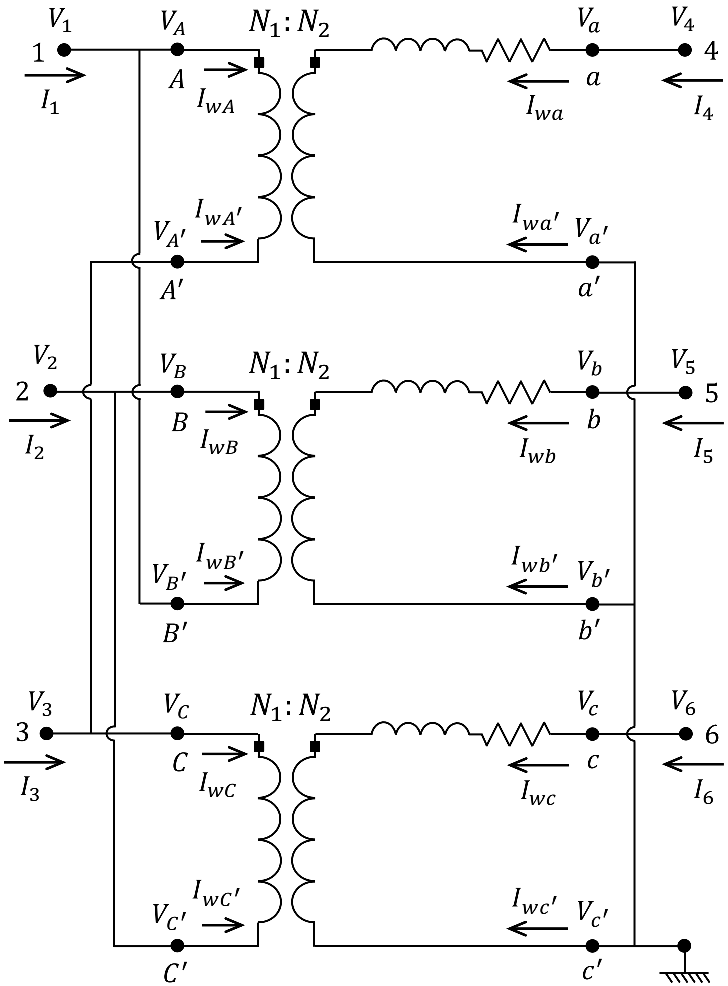
$$\begin{bmatrix} 0.00011343 - j0.00011343 & -0.00011343 + j0.00011343 & -0.00181039 + j0.00181039 & 0.00181039 - j0.00181039 \\ -0.00011343 + j0.00011343 & 0.00011343 - j0.00011343 & 0.00181039 - j0.00181039 & -0.00181039 + j0.00181039 \\ -0.00181039 + j0.00181039 & 0.00181039 - j0.00181039 & 0.02889238 - j0.02889238 & -0.02889238 + j0.02889238 \\ 0.00181039 - j0.00181039 & -0.00181039 + j0.00181039 & -0.02889238 + j0.02889238 & 0.02889238 - j0.02889238 \end{bmatrix}$$

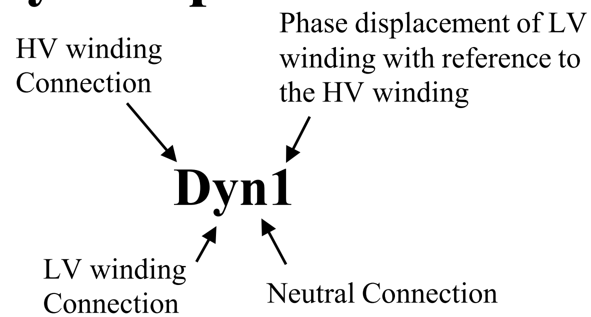
Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

						G	B
0.000113	-0.00011	-0.00011	0.000113	-0.00181	0.00181	0.00181	-0.00181
-0.00011	0.000113	0.000113	-0.00011	0.00181	-0.00181	-0.00181	0.00181
-0.00181	0.00181	0.00181	-0.00181	0.028892	-0.02889	-0.02889	0.028892
0.00181	-0.00181	-0.00181	0.00181	-0.02889	0.028892	0.028892	-0.02889

Modeling A Transformer in OpenDSS

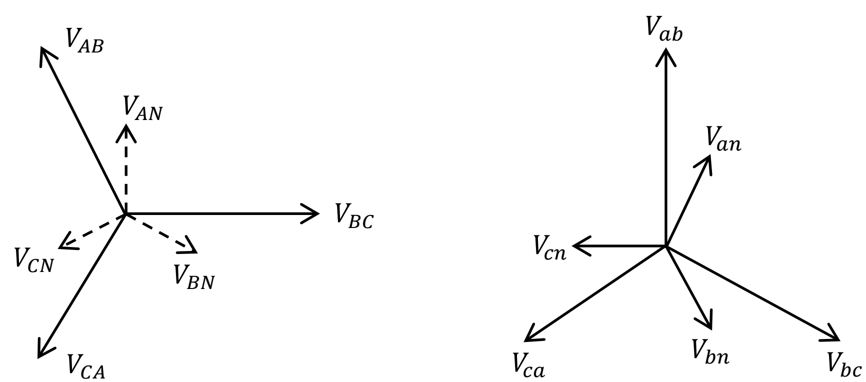
2. Three-phase Delta-Grounded Wye Step-down Transformer





Note that

- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

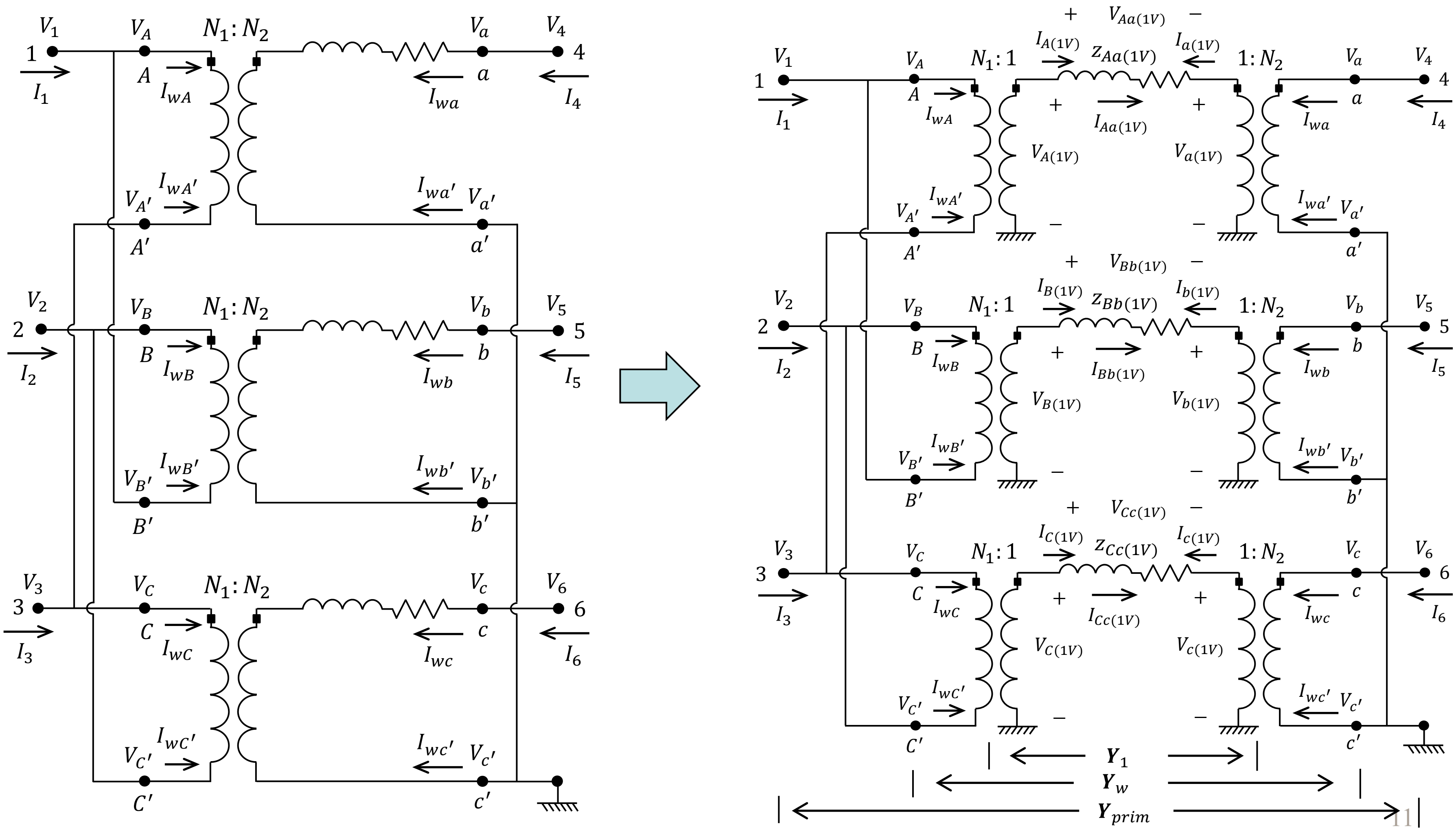


Dyn1

https://en.wikipedia.org/wiki/Vector_group

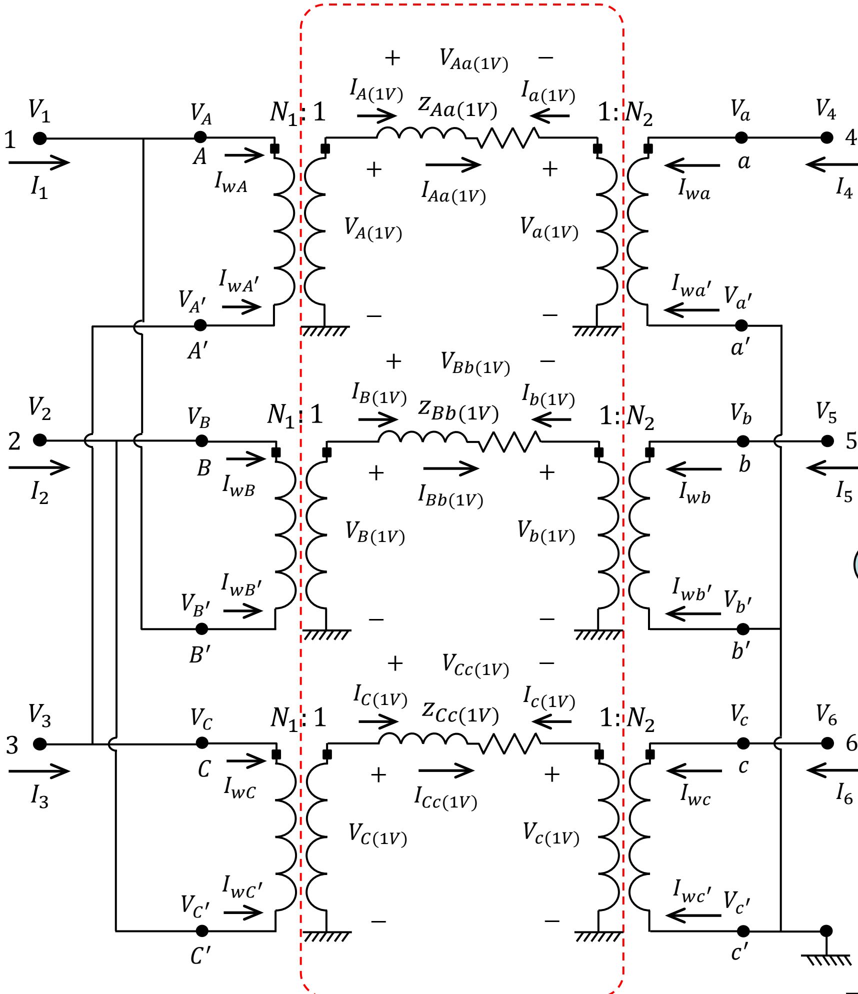
Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.

(1)
$$\begin{cases} I_{A(1V)} = I_{Aa(1V)} \\ I_{a(1V)} = -I_{Aa(1V)} \\ I_{B(1V)} = I_{Bb(1V)} \\ I_{b(1V)} = -I_{Bb(1V)} \\ I_{C(1V)} = I_{Cc(1V)} \\ I_{c(1V)} = -I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_1 = \boldsymbol{B}\boldsymbol{I}_{sc(1V)}, \text{ where, } \boldsymbol{I}_1 = \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \boldsymbol{I}_{sc(1V)} = \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

(2)
$$\begin{cases} V_{Aa(1V)} = V_{A(1V)} - V_{a(1V)} \\ V_{Bb(1V)} = V_{B(1V)} - V_{b(1V)} \\ V_{Cc(1V)} = V_{C(1V)} - V_{c(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix}$$

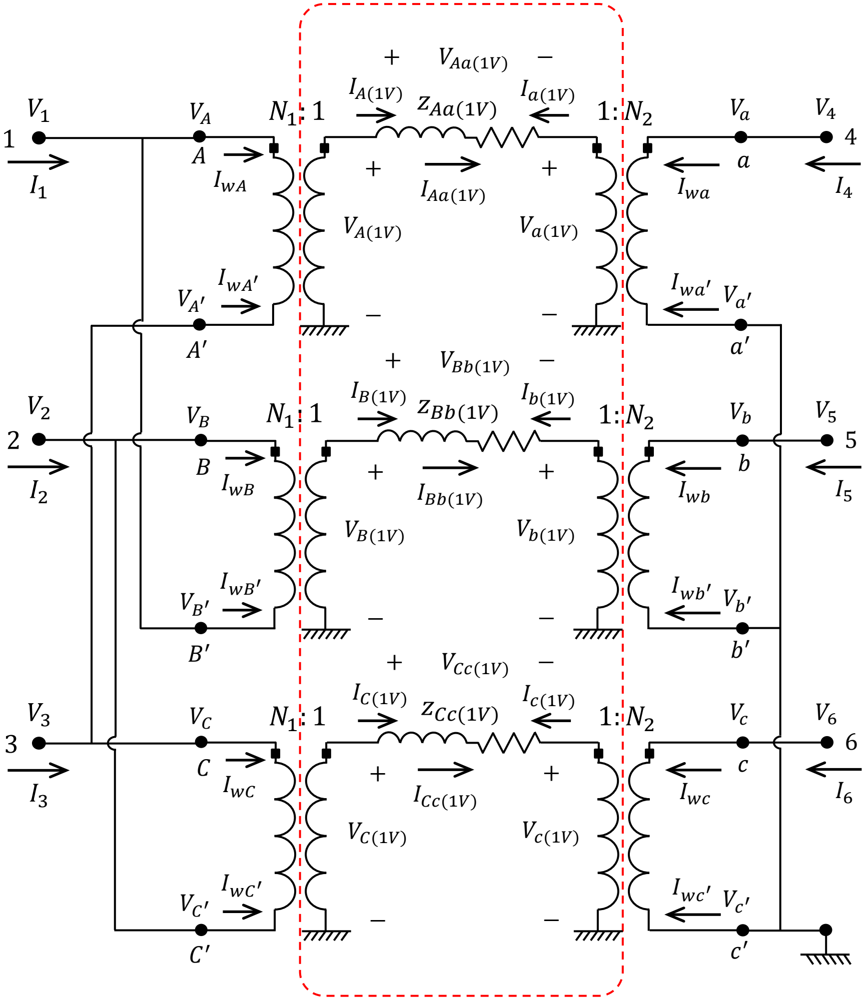
$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{B}^T\boldsymbol{V}_1, \text{ where, } \boldsymbol{V}_{sc(1V)} = \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix}, \boldsymbol{V}_1 = \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ \dot{V}_{c(1V)} \end{bmatrix}$$

(3)
$$\begin{cases} V_{Aa(1V)} = z_{Aa(1V)} I_{Aa(1V)} \\ V_{Bb(1V)} = z_{Bb(1V)} I_{Bb(1V)} \\ V_{Cc(1V)} = z_{Cc(1V)} I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} z_{Aa(1V)} & 0 & 0 \\ 0 & z_{Bb(1V)} & 0 \\ 0 & 0 & z_{Cc(1V)} \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{Z}_{sc(1V)}\boldsymbol{I}_{sc(1V)} \Rightarrow \boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} = \boldsymbol{I}_{sc(1V)}, \quad \text{where, } \boldsymbol{Y}_{sc(1V)} = (\boldsymbol{Z}_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Let $\boldsymbol{I}_1 = \boldsymbol{Y}_1\boldsymbol{V}_1$, where, $\boldsymbol{Y}_1$ is the admittance matrix on a one turn or one voltage base. What is $\boldsymbol{Y}_1$?

$$\left.\begin{aligned} \boldsymbol{I}_1 &= \boldsymbol{Y}_1\boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{B}\boldsymbol{I}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{B}\boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1\boldsymbol{V}_1 \\ \boldsymbol{V}_{sc(1V)} &= \boldsymbol{B}^T\boldsymbol{V}_1 \end{aligned}\right\}$$

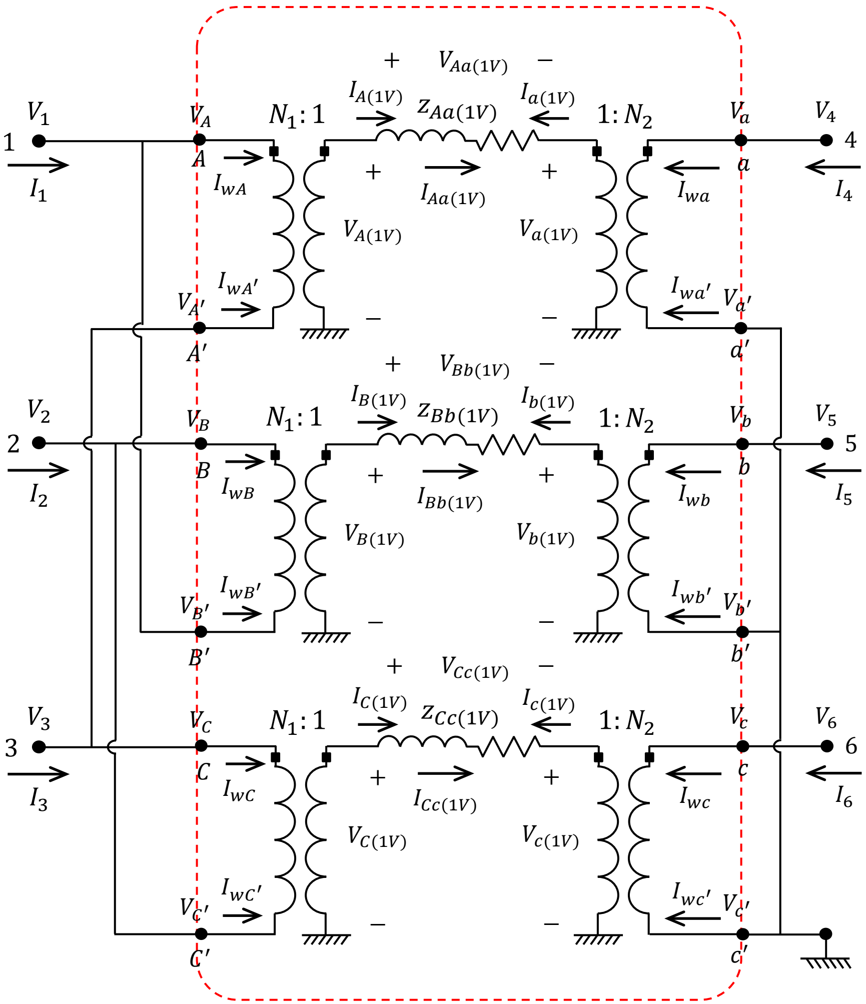
$$\Rightarrow \left.\begin{aligned} \boldsymbol{B}\boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1\left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1}\boldsymbol{V}_{sc(1V)} \\ \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} = \boldsymbol{Y}_1\left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1}\boldsymbol{V}_{sc(1V)}$$

$$\Rightarrow \boldsymbol{Y}_1 = \boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



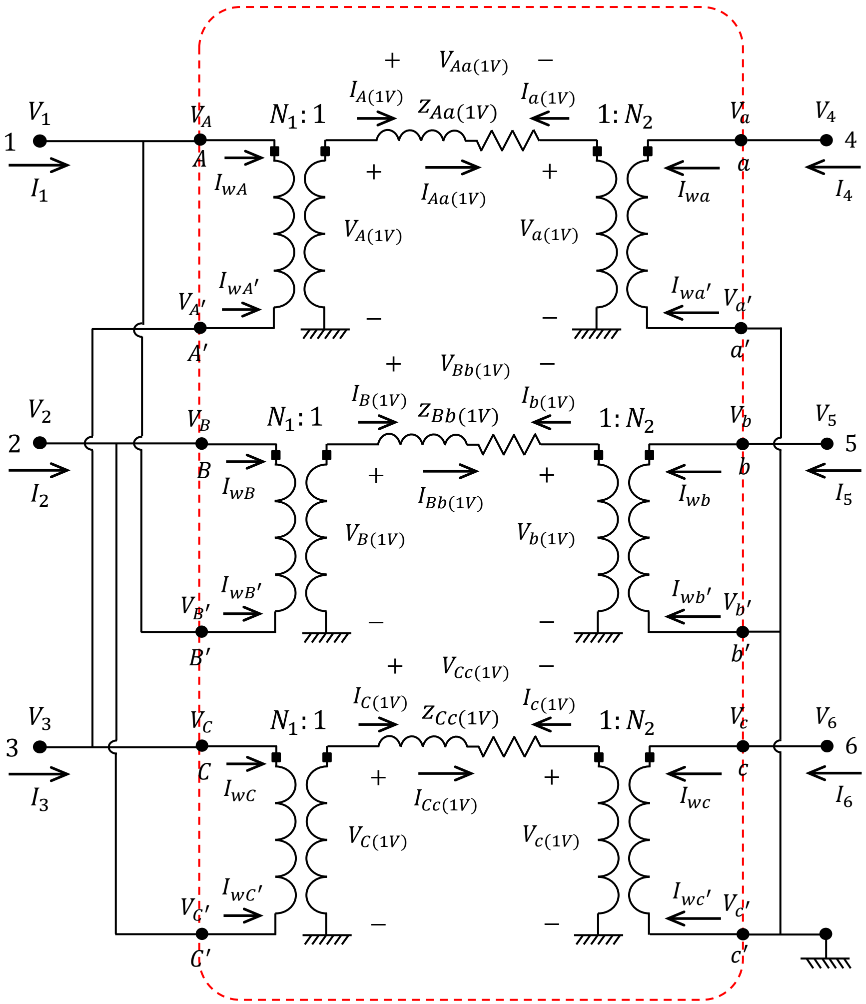
Step 2: Consider winding turns ratio.

①
$$\begin{cases} I_{wA} = I_{A(1V)}/N_1 \\ I_{wA'} = -I_{A(1V)}/N_1 \\ I_{wa} = I_{a(1V)}/N_2 \\ I_{wa'} = -I_{a(1V)}/N_2 \\ I_{wB} = I_{B(1V)}/N_1 \\ I_{wB'} = -I_{B(1V)}/N_1 \\ I_{wb} = I_{b(1V)}/N_2 \\ I_{wb'} = -I_{b(1V)}/N_2 \\ I_{wC} = I_{C(1V)}/N_1 \\ I_{wC'} = -I_{C(1V)}/N_1 \\ I_{wc} = I_{c(1V)}/N_2 \\ I_{wc'} = -I_{c(1V)}/N_2 \end{cases} \Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_w = \boldsymbol{N}\boldsymbol{I}_1, \text{ where } \boldsymbol{I}_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, \ \boldsymbol{N} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



(2)
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$\Rightarrow$

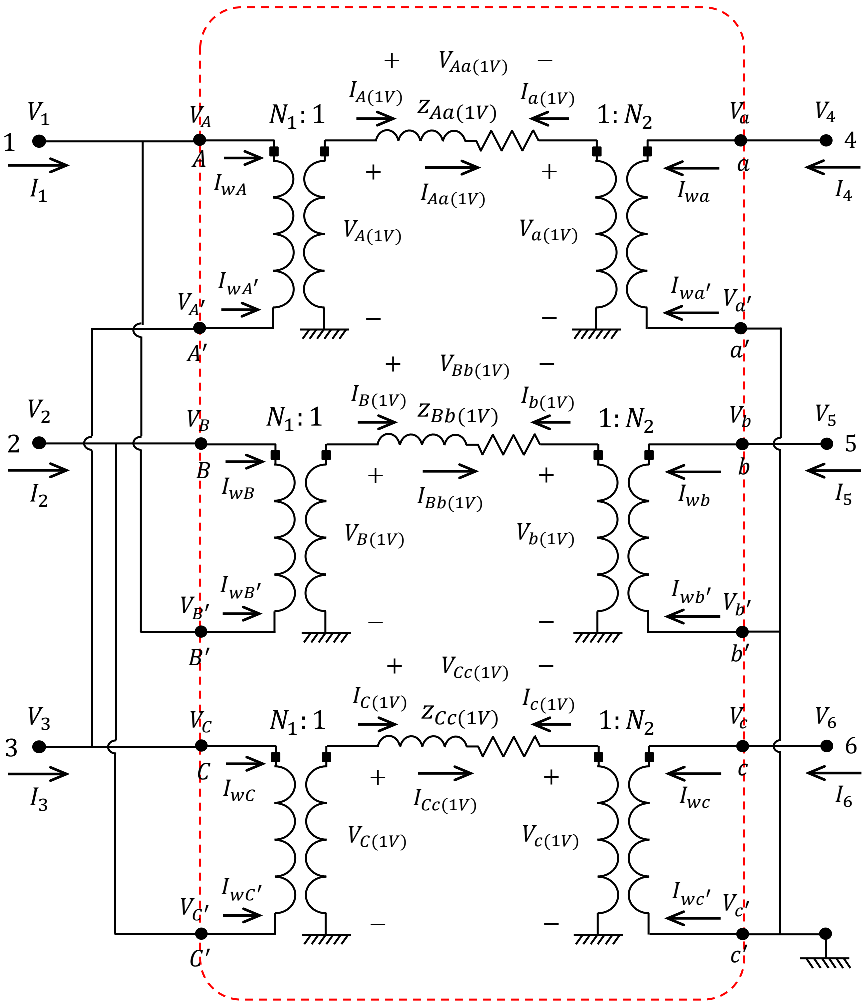
$$\begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$\Rightarrow$ $\boldsymbol{V}_1 = \boldsymbol{N}^T \boldsymbol{V}_w$, where, $\boldsymbol{V}_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$

Note: For a delta-connected winding, $N = VLL$; for a Y-connected winding, $N = \frac{VLL}{\sqrt{3}}$.

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Next, let $\boldsymbol{I}_w = \boldsymbol{Y}_w \boldsymbol{V}_w$. How to obtain $\boldsymbol{Y}_w$?

$$\left.\begin{aligned} \boldsymbol{I}_w &= \boldsymbol{Y}_w \boldsymbol{V}_w \\ \boldsymbol{I}_w &= \boldsymbol{N}\boldsymbol{I}_1 \\ \boldsymbol{V}_1 &= \boldsymbol{N}^T \boldsymbol{V}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{N}\boldsymbol{I}_1 &= \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \end{aligned}\right\}$$

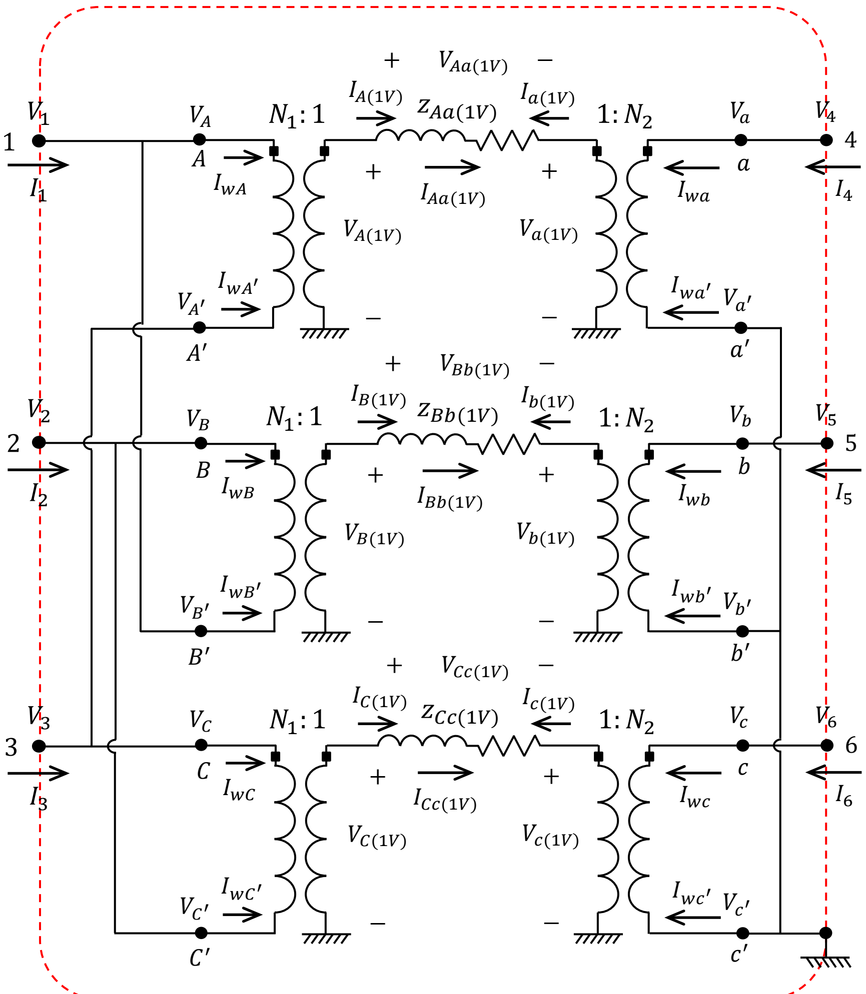
$$\Rightarrow \boldsymbol{N}\boldsymbol{Y}_1 \boldsymbol{V}_1 = \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{Y}_1 \boldsymbol{N}^T \\ \boldsymbol{Y}_1 &= \boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_w = \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{N}^T$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Step 3: Consider winding connections

(1)
$$\begin{cases} I_1 = I_{wA} + I_{wB'} \\ I_2 = I_{wB} + I_{wC'} \\ I_3 = I_{wC} + I_{wA'} \\ I_4 = I_{wa} \\ I_5 = I_{wb} \\ I_6 = I_{wc} \end{cases}$$

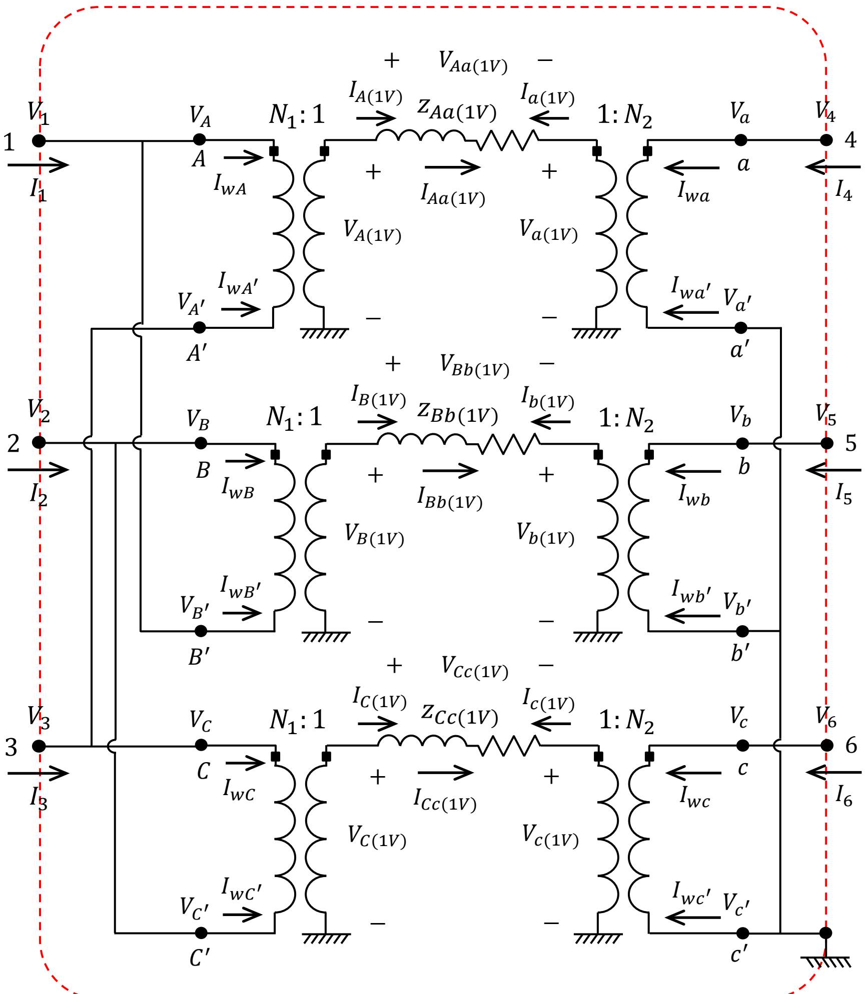
$\Rightarrow$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$\Rightarrow$ $\boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_w$, where, $\boldsymbol{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$, $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



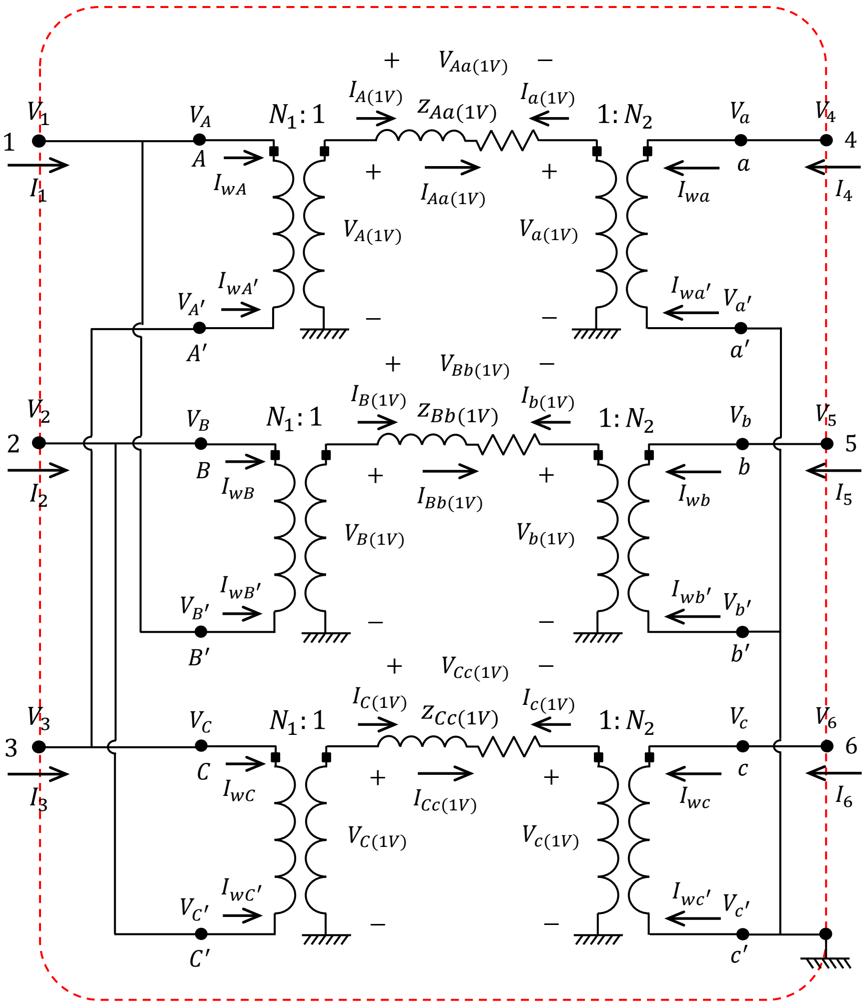
(2)

$$
\begin{cases}
V_A = V_1 \\
V_{A'} = V_3 \\
V_a = V_4 \\
V_{a'} = 0 \\
V_B = V_2 \\
V_{B'} = V_1 \\
V_b = V_5 \\
V_{b'} = 0 \\
V_C = V_3 \\
V_{C'} = V_2 \\
V_c = V_6 \\
V_{c'} = 0
\end{cases}
\Rightarrow
\begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}
$$

$\Rightarrow$ $\boldsymbol{V}_w = \boldsymbol{A}^T \boldsymbol{V}_{prim}$, where, $\boldsymbol{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$.

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer



Let $\boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim}$. How to obtain $\boldsymbol{Y}_{prim}$?

$$\left.\begin{aligned}\boldsymbol{I}_{prim} &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim}\\ \boldsymbol{I}_{prim} &= \boldsymbol{A}\boldsymbol{I}_{w}\end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned}\boldsymbol{A}\boldsymbol{I}_{w} &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim}\\ \boldsymbol{V}_{w} &= \boldsymbol{A}^{T}\,\boldsymbol{V}_{prim}\end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned}\boldsymbol{A}\boldsymbol{I}_{w} &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w}\\ \boldsymbol{I}_{w} &= \boldsymbol{Y}_{w}\,\boldsymbol{V}_{w}\end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned}\boldsymbol{A}\boldsymbol{Y}_{w}\,\boldsymbol{V}_{w} &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w}\\ \boldsymbol{Y}_{w} &= \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_{prim} = \boldsymbol{A}\boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\,\boldsymbol{A}^{T}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3     conn=delta kV=115    kva=500 %r=1
~ wdg=2 bus=M.1.2.3.0   conn=wye    kV=4.16  kva=500 %r=1
```

Transformer name

Number of phases

Number of windings

Percent reactance high-to-low

Specify which winding will be edited

Winding connection

Rated voltage

Base kVA rating

Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184\ S$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

2. Three-phase Delta-Grounded Wye Step-down Transformer

$$\boldsymbol{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANBY}_{sc(1V)}\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\ \boldsymbol{A}^{T}$=

$$\begin{bmatrix} 0.00017383 - j0.00043457 & -8.6913e-05 + j0.00021728 & -8.6913e-05 + j0.00021728 & -0.00416150 + j0.01040376 & 0.00416150 - j0.01040376 & 0 + j0 \\ -8.6913e-05 + j0.00021728 & 0.00017383 - j0.00043457 & -8.6913e-05 + j0.00021728 & 0 + j0 & -0.00416150 + j0.01040376 & 0.00416150 - j0.01040376 \\ -8.6913e-05 + j0.00021728 & -8.6913e-05 + j0.00021728 & 0.00017383 - j0.00043457 & 0.00416150 - j0.01040376 & 0 + j0 & -0.00416150 + j0.01040376 \\ -0.00416150 + j0.01040376 & 0 + j0 & 0.000260739 - \mathrm{j}0.00065184 & 0.19925780 - j0.49814451 & 0 + j0 & 0 + j0 \\ 0.00416150 - \mathrm{j}0.01040375 & -0.00416150 + j0.01040376 & 0 + j0 & 0 + j0 & 0.19925780 - j0.49814451 & 0 + j0 \\ 0 + j0 & 0.00416150 - j0.01040376 & -0.00416150 + j0.01040376 & 0 + j0 & 0 + j0 & 0.19925780 - j0.49814451 \end{bmatrix}$$

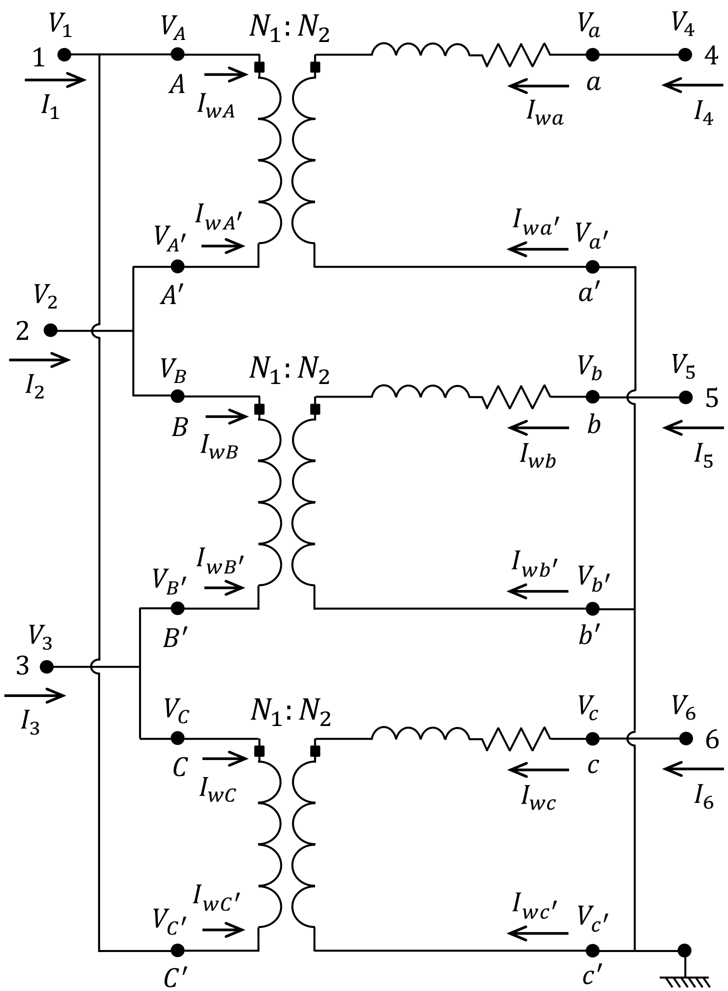
Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

G ↓ B ↓

	1		2		3		4		5		6	
1	0.000174	-0.00043	-8.69E-05	0.000217	-8.69E-05	0.000217	-0.00416	0.010404	0.004162	-0.0104	0	0
2	-8.69E-05	0.000217	0.000174	-0.00043	-8.69E-05	0.000217	0	0	-0.00416	0.010404	0.004162	-0.0104
3	-8.69E-05	0.000217	-8.69E-05	0.000217	0.000174	-0.00043	0.004162	-0.0104	0	0	-0.00416	0.010404
4	-0.00416	0.010404	0	0	0.004162	-0.0104	0.199258	-0.49814	0	0	0	0
5	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0	0.199258	-0.49814	0	0
6	0	0	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0	0.199258	-0.49814

Modeling A Transformer in OpenDSS

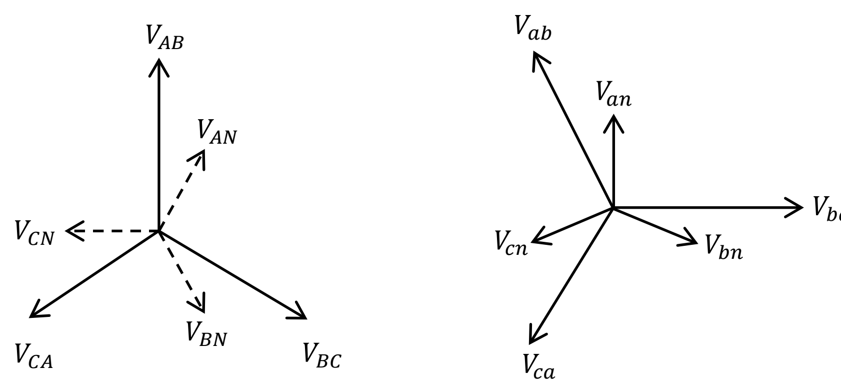
3. Three-phase Delta-Grounded Wye Step-up Transformer



As the IEC60076-1 standard has stated, the notation is **HV-LV** in sequence. Therefore, this step-up transformer with a delta-connected primary, and star-connected secondary, is not written as 'dY1', but 'Yd1'. The 1 indicates the LV winding lags the HV by 30 degrees.

Note that

- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

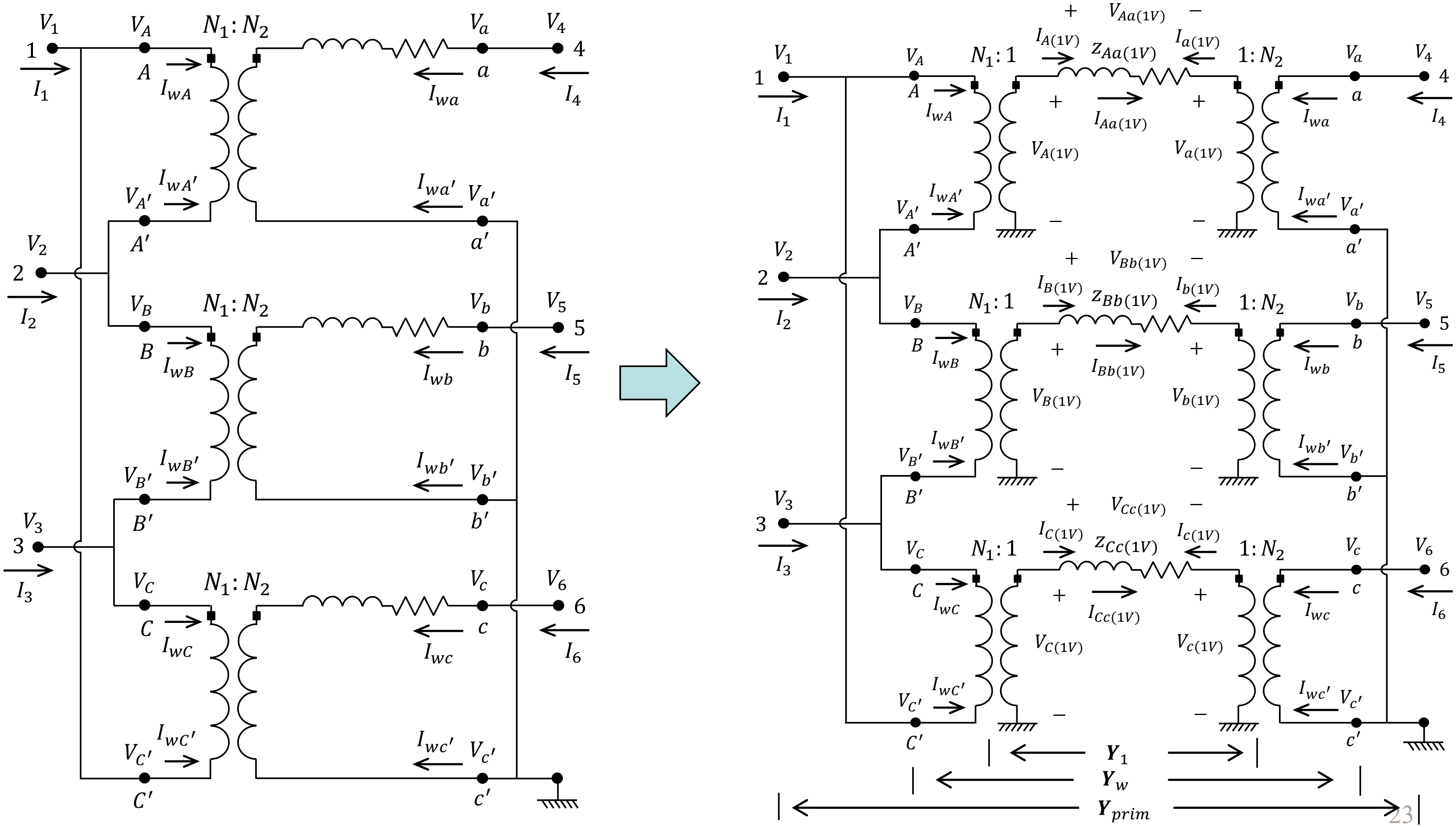


YNd1

https://en.wikipedia.org/wiki/Vector_group

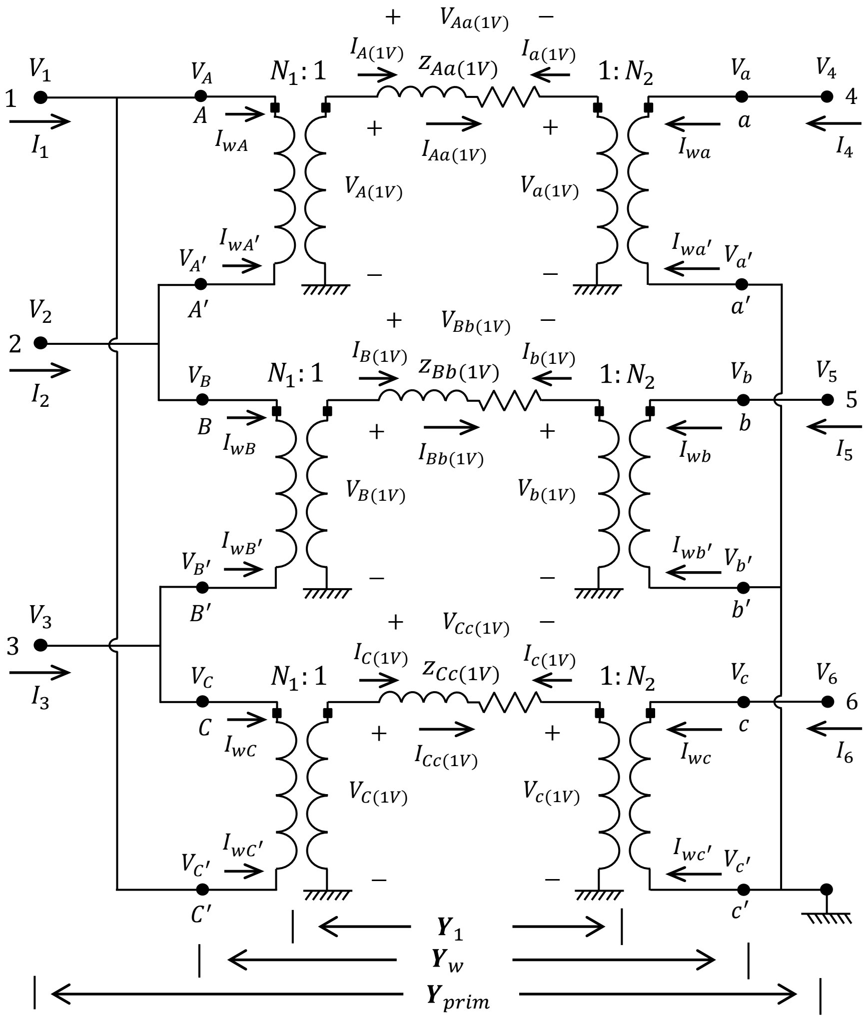
Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer



Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer



Note:

Compared to the 3-phase delta-grounded wye step-down transformer, the equation, $\boldsymbol{Y}_{prim} = \boldsymbol{ANB}y_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\,\boldsymbol{A}^{T}$, still applies to a 3-phase delta-grounded wye step-up transformer. The only difference exists in the formation of matrix $\boldsymbol{A}$ due to a different connection of delta winding, and the values of N_1 and N_2. Specifically, according to the figure on the left, we have

$$\begin{cases} I_1 = I_{wA} + I_{wC'} \\ I_2 = I_{wB} + I_{wA'} \\ I_3 = I_{wC} + I_{wB'} \\ I_4 = I_{wa} \\ I_5 = I_{wb} \\ I_6 = I_{wc} \end{cases} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3   conn=delta kV=4.16  kva=500 %r=1
~ wdg=2 bus=M.1.2.3.0 conn=wye   kV=115   kva=500 %r=1
```

Transformer name

Number of phases

Number of windings

Percent reactance high-to-low

Specify which winding will be edited

Winding connection

Rated voltage

Base kVA rating

Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

3. Three-phase Delta-Grounded Wye Step-up Transformer

$$\boldsymbol{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANBY}_{sc(1V)}\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\ \boldsymbol{A}^{T}$ =

$$\begin{bmatrix} 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 & -0.00416150 + j\,0.01040376 & 0 + j0 & 0.00416150 - j0.01040376 \\ -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 & 0.00416150 - j0.01040376 & -0.00416150 + j\,0.01040376 & 0 + j0 \\ -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 & 0 + j0 & 0.00416150 - j0.01040376 & -0.00416150 + j\,0.01040376 \\ -0.00416150 + j\,0.01040376 & 0.00416150 - j0.01040376 & 0 + j0 & 0.00026074 - j0.00065185 & 0 + j0 & 0 + j0 \\ 0 + j0 & -0.00416150 + j\,0.01040376 & 0.00416150 - j0.01040376 & 0 + j0 & 0.00026074 - j0.00065185 & 0 + j0 \\ 0.00416150 - j0.01040376 & 0 + j0 & -0.00416150 + j\,0.01040376 & 0 + j0 & 0 + j0 & 0.00026074 - j0.00065185 \end{bmatrix}$$

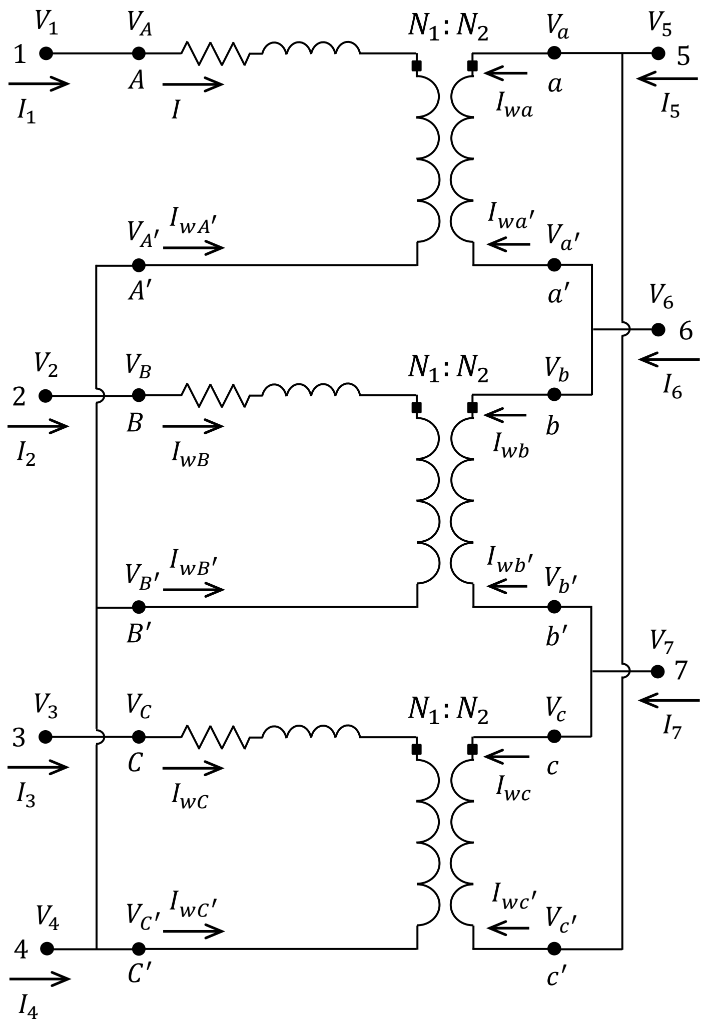
Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

G ↓ B ↓

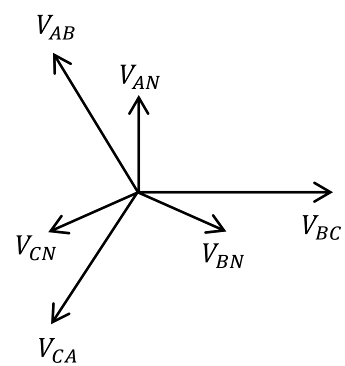
	1 (G)	1 (B)	2 (G)	2 (B)	3 (G)	3 (B)	4 (G)	4 (B)	5 (G)	5 (B)	6 (G)	6 (B)
1	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048	-0.00416	0.010404	0	0	0.004162	-0.0104
2	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048	0.004162	-0.0104	-0.00416	0.010404	0	0
3	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321	0	0	0.004162	-0.0104	-0.00416	0.010404
4	-0.00416	0.010404	0.004162	-0.0104	0	0	0.000261	-0.00065	0	0	0	0
5	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0	0.000261	-0.00065	0	0
6	0.004162	-0.0104	0	0	-0.00416	0.010404	0	0	0	0	0.000261	-0.00065

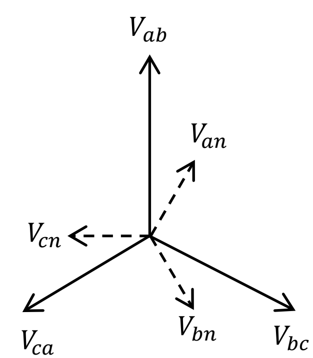
Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

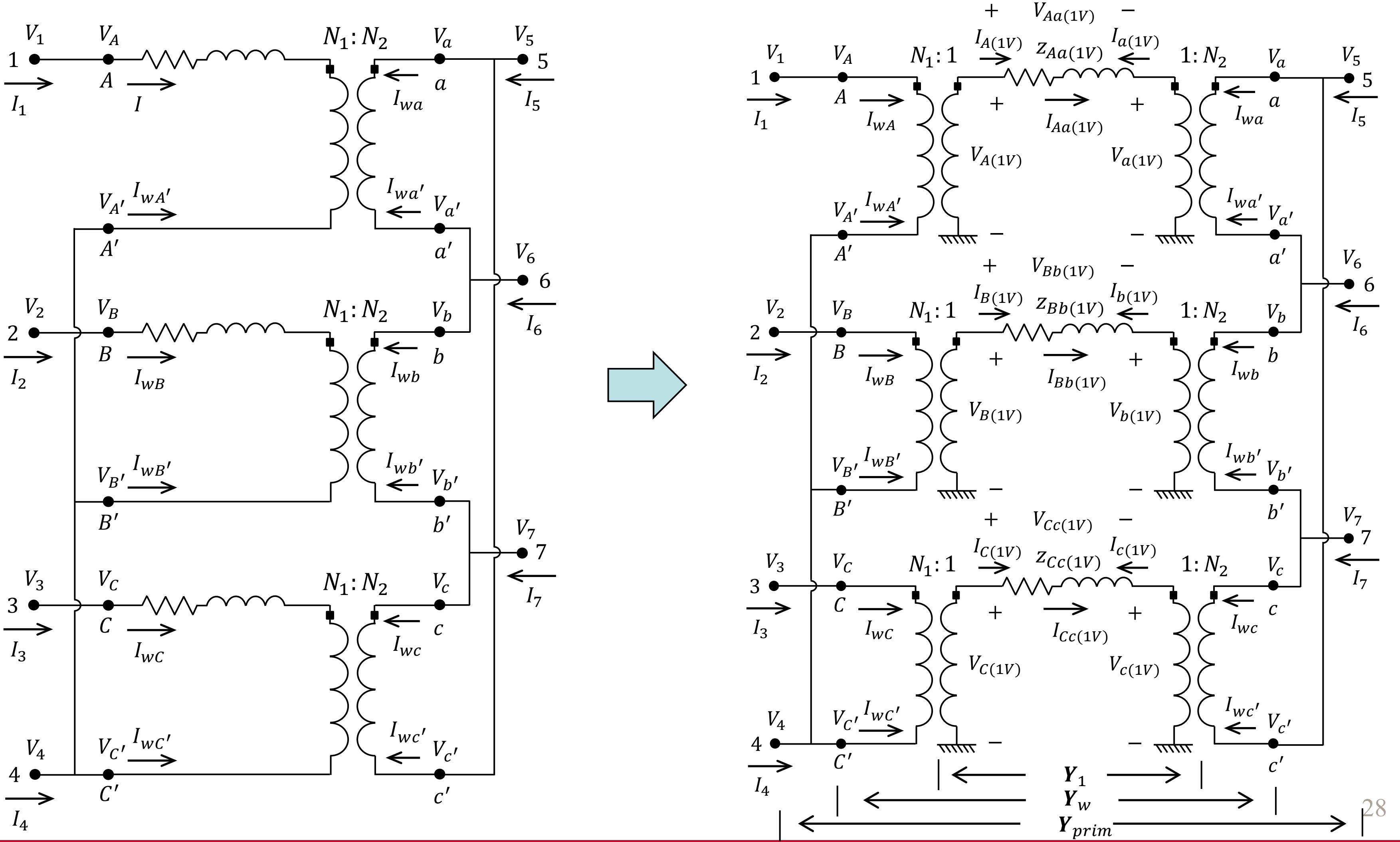




YNd1

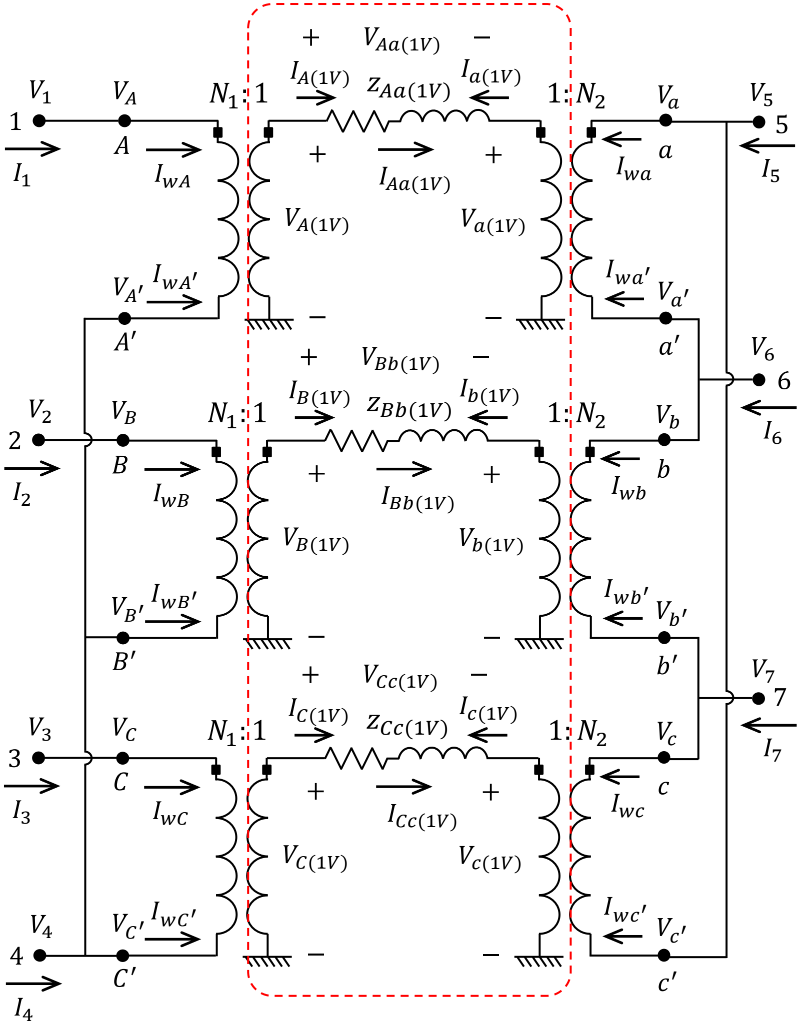
Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.

①
$$\begin{cases} I_{A(1V)} = I_{Aa(1V)} \\ I_{a(1V)} = -I_{Aa(1V)} \\ I_{B(1V)} = I_{Bb(1V)} \\ I_{b(1V)} = -I_{Bb(1V)} \\ I_{C(1V)} = I_{Cc(1V)} \\ I_{c(1V)} = -I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_1 = \boldsymbol{B}\boldsymbol{I}_{sc(1V)}, \text{ where, } \boldsymbol{I}_1 = \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \boldsymbol{I}_{sc(1V)} = \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

②
$$\begin{cases} V_{Aa(1V)} = V_{A(1V)} - V_{a(1V)} \\ V_{Bb(1V)} = V_{B(1V)} - V_{b(1V)} \\ V_{Cc(1V)} = V_{C(1V)} - V_{c(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix}$$

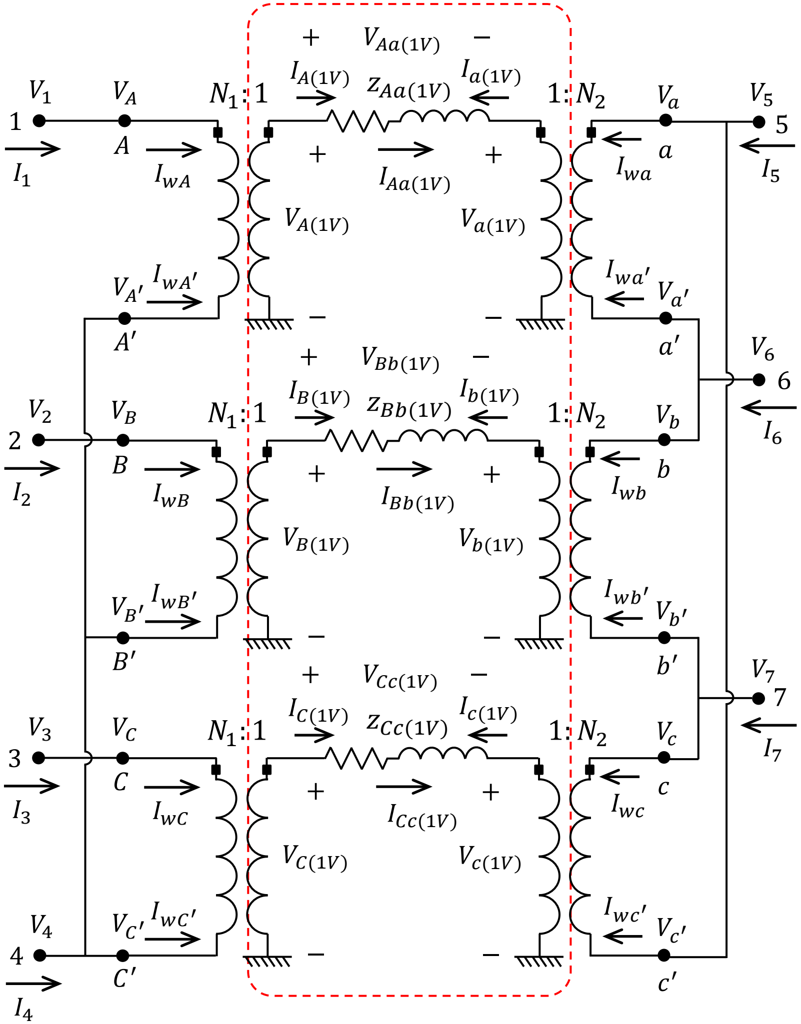
$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{B}^T\boldsymbol{V}_1, \text{ where, } \boldsymbol{V}_{sc(1V)} = \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix}, \boldsymbol{V}_1 = \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix}$$

③
$$\begin{cases} V_{Aa(1V)} = z_{Aa(1V)} I_{Aa(1V)} \\ V_{Bb(1V)} = z_{Bb(1V)} I_{Bb(1V)} \\ V_{Cc(1V)} = z_{Cc(1V)} I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} z_{Aa(1V)} & 0 & 0 \\ 0 & z_{Bb(1V)} & 0 \\ 0 & 0 & z_{Cc(1V)} \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{Z}_{sc(1V)}\boldsymbol{I}_{sc(1V)} \Rightarrow \boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} = \boldsymbol{I}_{sc(1V)}, \quad where, \boldsymbol{Y}_{sc(1V)} = (\boldsymbol{Z}_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Let $\boldsymbol{I}_1 = \boldsymbol{Y}_1\boldsymbol{V}_1$, where, $\boldsymbol{Y}_1$ is the admittance matrix on a one turn or one voltage base. What is $\boldsymbol{Y}_1$?

$$\left.\begin{aligned} \boldsymbol{I}_1 &= \boldsymbol{Y}_1\boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{B}\boldsymbol{I}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{B}\boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1\boldsymbol{V}_1 \\ \boldsymbol{V}_{sc(1V)} &= \boldsymbol{B}^T\boldsymbol{V}_1 \end{aligned}\right\}$$

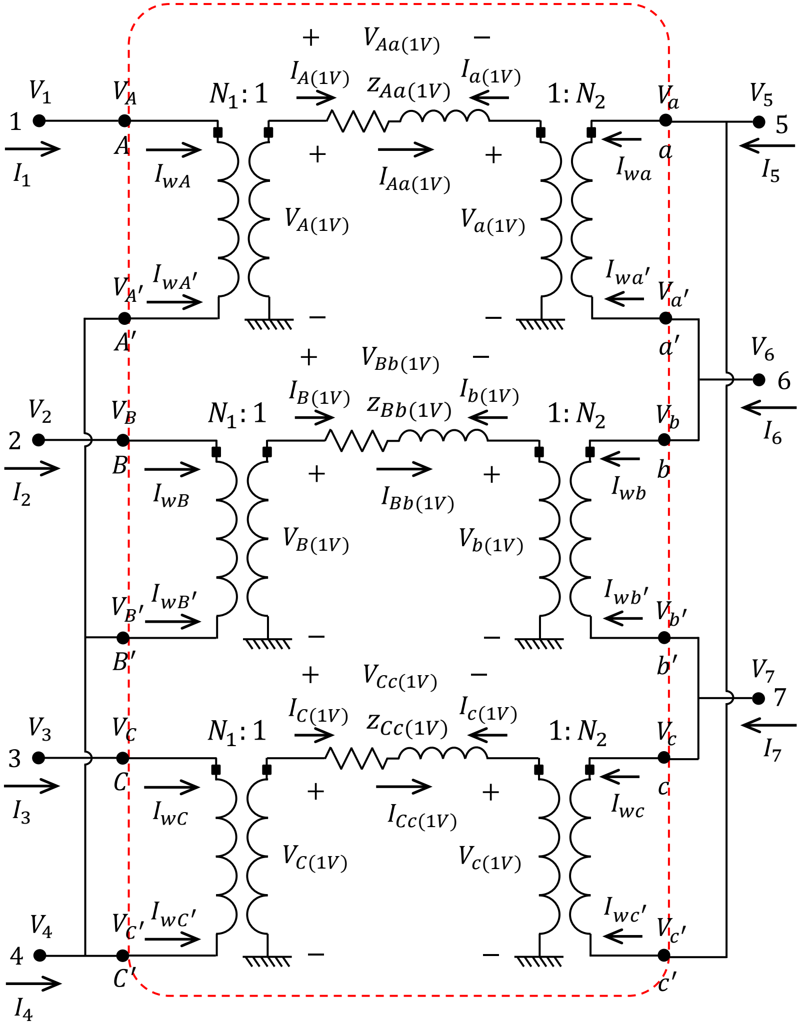
$$\Rightarrow \left.\begin{aligned} \boldsymbol{B}\boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1\left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1}\boldsymbol{V}_{sc(1V)} \\ \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} = \boldsymbol{Y}_1\left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1}\boldsymbol{V}_{sc(1V)}$$

$$\Rightarrow \boldsymbol{Y}_1 = \boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Step 2: Consider winding turns ratio.

①
$$\begin{cases} I_{wA} = I_{A(1V)}/N_1 \\ I_{wA'} = -I_{A(1V)}/N_1 \\ I_{wa} = I_{a(1V)}/N_2 \\ I_{wa'} = -I_{a(1V)}/N_2 \\ I_{wB} = I_{B(1V)}/N_1 \\ I_{wB'} = -I_{B(1V)}/N_1 \\ I_{wb} = I_{b(1V)}/N_2 \\ I_{wb'} = -I_{b(1V)}/N_2 \\ I_{wC} = I_{C(1V)}/N_1 \\ I_{wC'} = -I_{C(1V)}/N_1 \\ I_{wc} = I_{c(1V)}/N_2 \\ I_{wc'} = -I_{c(1V)}/N_2 \end{cases}$$

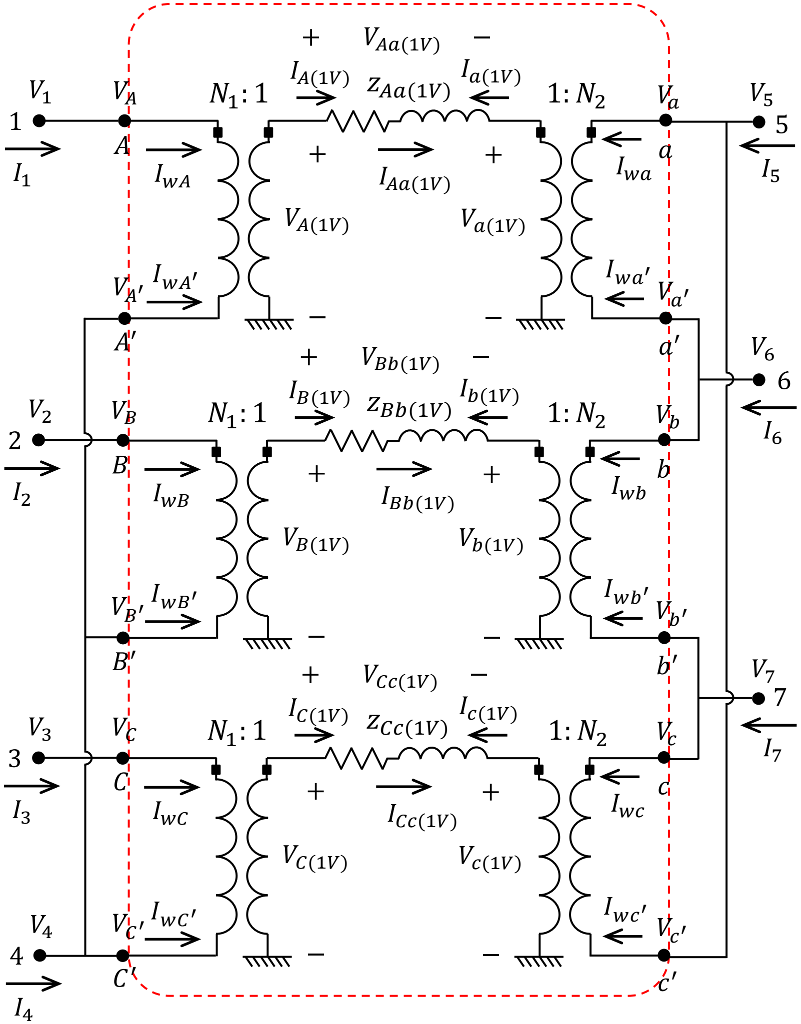
⇨

$$\begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

⇨ $\boldsymbol{I}_w = \boldsymbol{N}\boldsymbol{I}_1$, where $\boldsymbol{I}_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$, $\boldsymbol{N} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



(2)

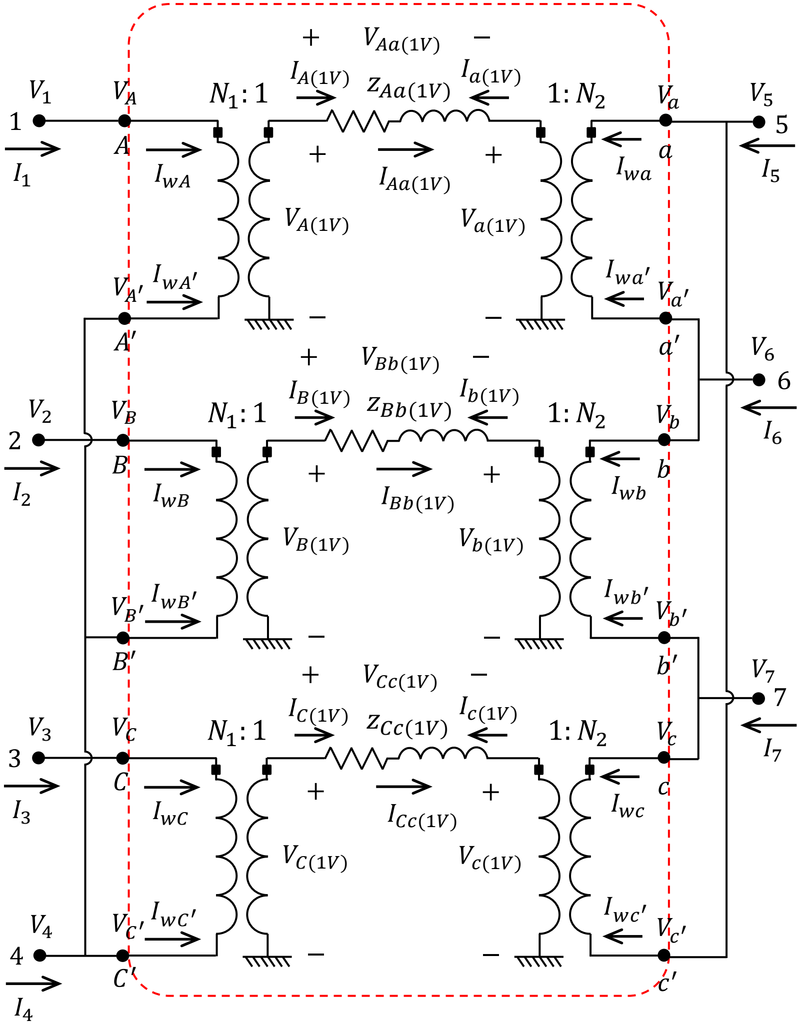
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_1 = \boldsymbol{N}^T \boldsymbol{V}_w, \text{ where, } \boldsymbol{v}_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Next, let $\boldsymbol{I}_w = \boldsymbol{Y}_w \boldsymbol{V}_w$. How to obtain $\boldsymbol{Y}_w$?

$$\left.\begin{aligned} \boldsymbol{I}_w &= \boldsymbol{Y}_w \boldsymbol{V}_w \\ \boldsymbol{I}_w &= \boldsymbol{N}\boldsymbol{I}_1 \\ \boldsymbol{V}_1 &= \boldsymbol{N}^T\boldsymbol{V}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{N}\boldsymbol{I}_1 &= \boldsymbol{Y}_w(\boldsymbol{N}^T)^{-1}\boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{Y}_1\boldsymbol{V}_1 \end{aligned}\right\}$$

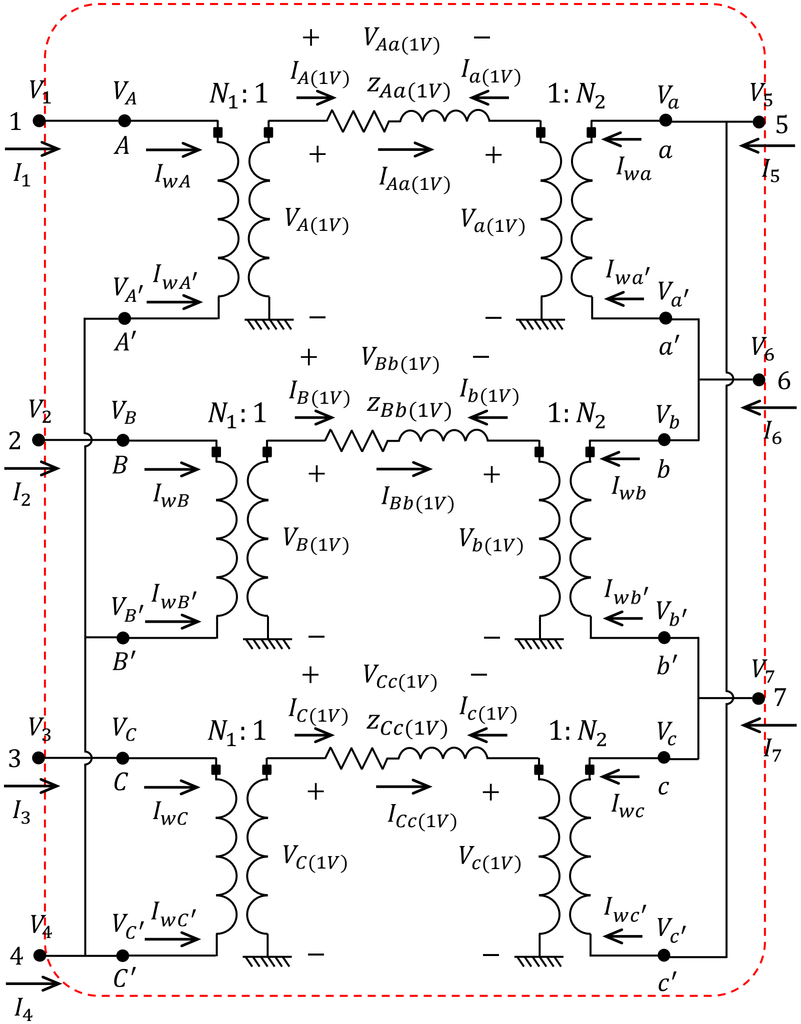
$$\Rightarrow \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{V}_1 = \boldsymbol{Y}_w(\boldsymbol{N}^T)^{-1}\boldsymbol{V}_1$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{N}^T \\ \boldsymbol{Y}_1 &= \boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{\mathrm{T}} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_w = \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Step 3: Consider winding connections

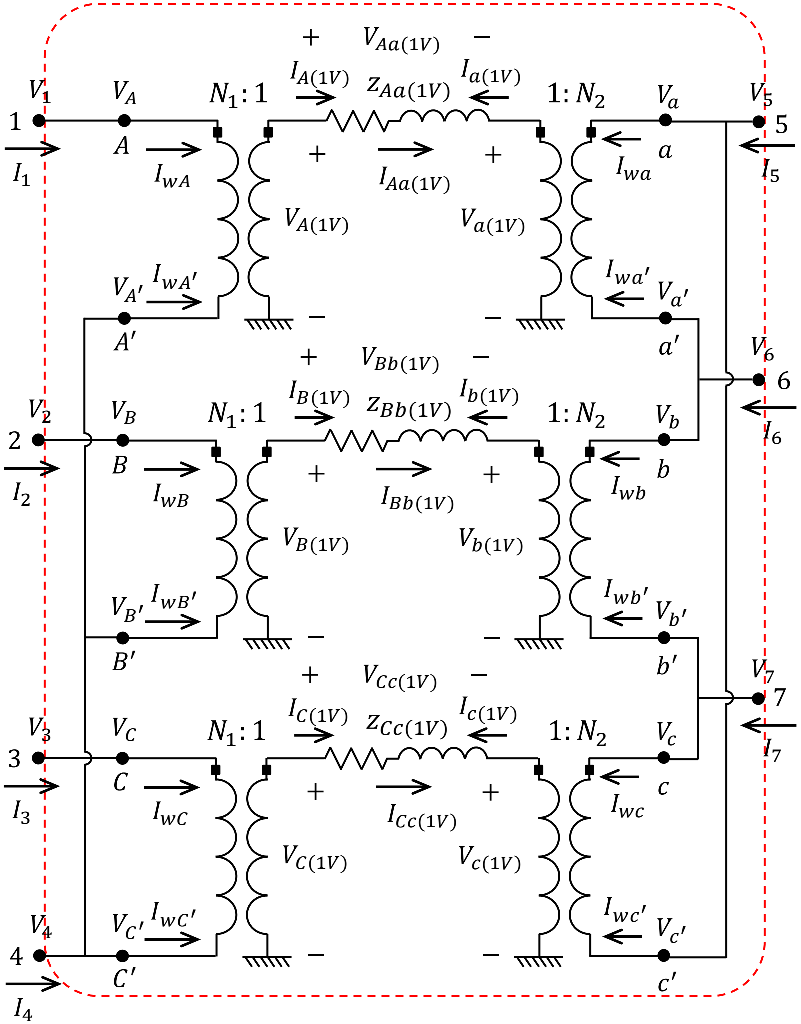
①
$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wA'} + I_{wB'} + I_{wC'} \\ I_5 = I_{wa} + I_{wc'} \\ I_6 = I_{wa'} + I_{wb} \\ I_7 = I_{wb'} + I_{wc} \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_w, \text{ where, } \boldsymbol{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

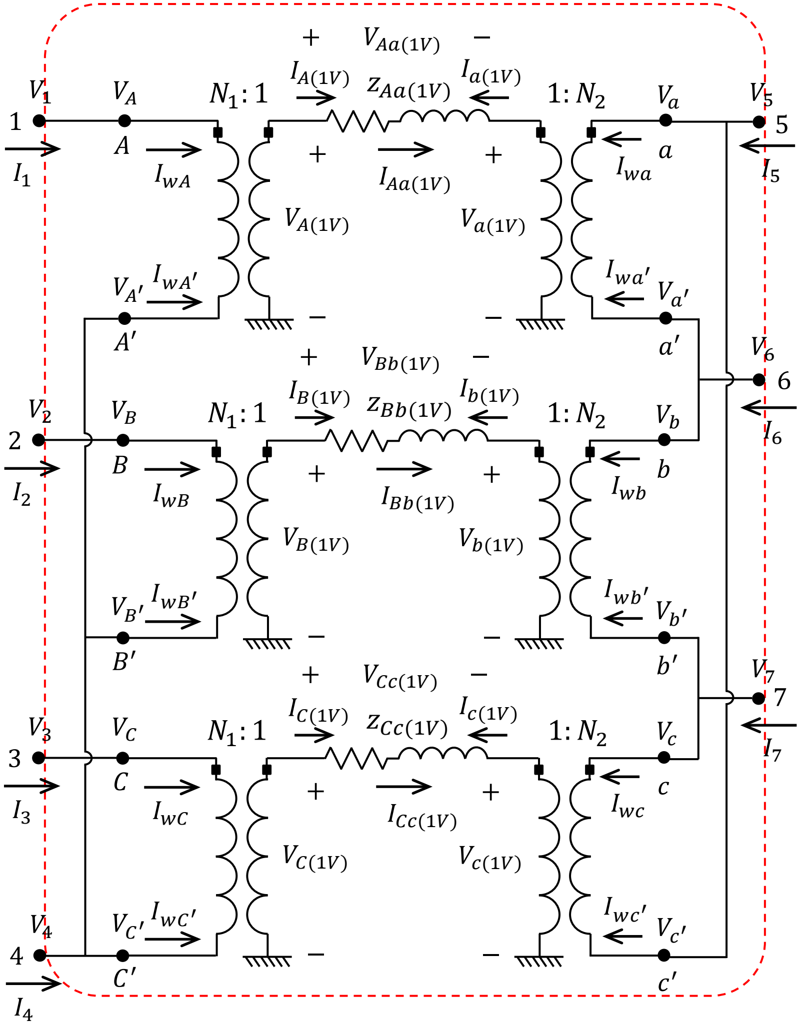


②
$$\begin{cases} V_A = V_1 \\ V_{A'} = V_4 \\ V_a = V_5 \\ V_{a'} = V_6 \\ V_B = V_2 \\ V_{B'} = V_4 \\ V_b = V_6 \\ V_{b'} = V_7 \\ V_C = V_3 \\ V_{C'} = V_4 \\ V_c = V_7 \\ V_{c'} = V_5 \end{cases} \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}$$

$\Rightarrow$ $\boldsymbol{V}_w = \boldsymbol{A}^T \boldsymbol{V}_{prim}$, where, $\boldsymbol{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}$.

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer



Let $\boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\boldsymbol{V}_{prim}$. How to obtain $\boldsymbol{Y}_{prim}$?

$$\left.\begin{array}{l} \boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\boldsymbol{V}_{prim} \\ \boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_{w} \end{array}\right\}$$

$$\Rightarrow \left.\begin{array}{l} \boldsymbol{A}\boldsymbol{I}_{w} = \boldsymbol{Y}_{prim}\boldsymbol{V}_{prim} \\ \boldsymbol{V}_{w} = \boldsymbol{A}^{T}\boldsymbol{V}_{prim} \end{array}\right\}$$

$$\Rightarrow \left.\begin{array}{l} \boldsymbol{A}\boldsymbol{I}_{w} = \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w} \\ \boldsymbol{I}_{w} = \boldsymbol{Y}_{w}\boldsymbol{V}_{w} \end{array}\right\}$$

$$\Rightarrow \left.\begin{array}{l} \boldsymbol{A}\boldsymbol{Y}_{w}\boldsymbol{V}_{w} = \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w} \\ \boldsymbol{Y}_{w} = \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{T}\boldsymbol{N}^{T} \end{array}\right\}$$

$$\Rightarrow \boldsymbol{Y}_{prim} = \boldsymbol{A}\boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{T}\boldsymbol{N}^{T}\boldsymbol{A}^{T}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3.4  conn=wye   kV=115   kva=500 %r=1
~ wdg=2 bus=M.1.2.3    conn=delta kV=4.16  kva=500 %r=1
```

Transformer name; Number of phases; Number of windings; Percent reactance high-to-low; Specify which winding will be edited; Winding connection; Rated voltage; Base kVA rating; Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184\ S$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

4. Three-phase Ungrounded Wye-Delta Step-down Transformer

$$\boldsymbol{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANBY}_{sc(1V)}\, \boldsymbol{B}^{\mathrm{T}} \boldsymbol{N}^{T}\, \boldsymbol{A}^{T}$ =

$$\begin{bmatrix}
0.000260739 - j0.00065184 & 0+j0 & 0+j0 & -0.000260739 + j0.00065184 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & 0+j0 \\
0+j0 & 0.000260739 - j0.00065184 & 0+j0 & -0.000260739 + j0.00065184 & 0+j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 \\
0+j0 & 0+j0 & 0.000260739 - j0.00065184 & -0.000260739 + j0.00065184 & 0.00416150 - j0.01040375 & 0+j0 & -0.00416150 + j0.01040375 \\
-0.000260739 + j0.00065184 & -0.000260739 + j0.00065184 & -0.000260739 + j0.00065184 & 0.00078221 - j0.00195554 & 0+j0 & 0+j0 & 0+j0 \\
-0.00416150 + j0.01040375 & 0+j0 & 0.00416150 - j0.01040375 & 0+j0 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 \\
0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0+j0 & 0+j0 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 \\
0+j0 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0+j0 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634
\end{bmatrix}$$

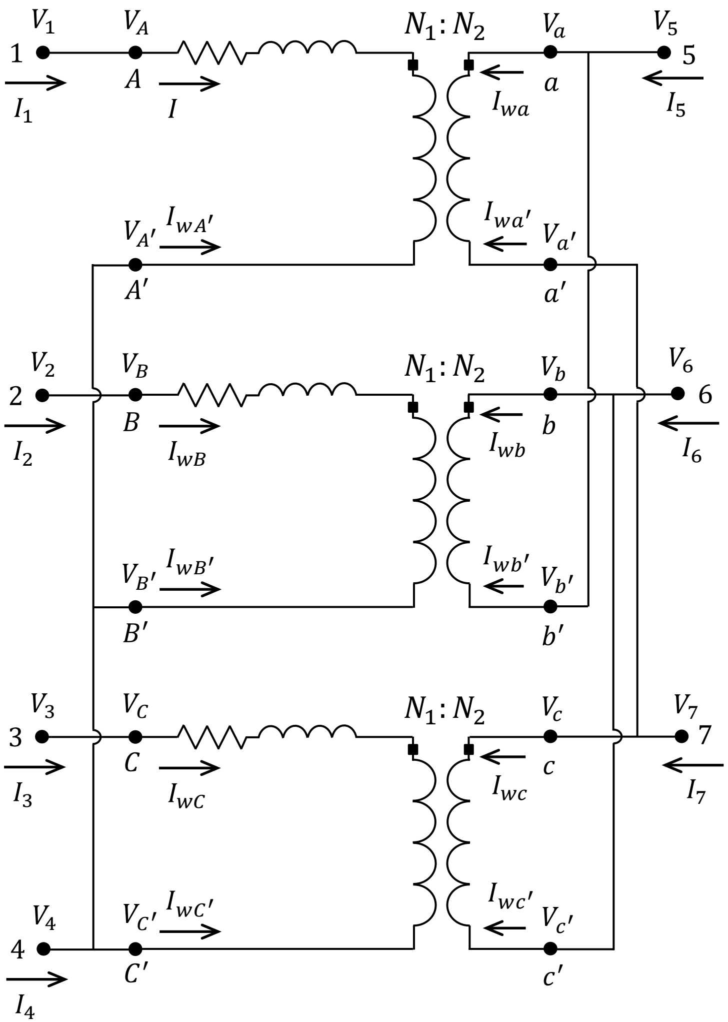
Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

							G	B								
1	0.000261	-0.00065	0	0	0	0	-0.00026	0.000652	-0.00416	0.010404	0.004162	-0.0104	0	0	0	0
2	0	0	0.000261	-0.00065	0	0	-0.00026	0.000652	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0
3	0	0	0	0	0.000261	-0.00065	-0.00026	0.000652	0.004162	-0.0104	0	0	-0.00416	0.010404	0	0
4	-0.00026	0.000652	-0.00026	0.000652	-0.00026	0.000652	0.000782	-0.00196	0	0	0	0	0	0	0	0
5	-0.00416	0.010404	0	0	0.004162	-0.0104	0	0	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048	0	0
6	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048	0	0
7	0	0	0.004162	-0.0104	-0.00416	0.010404	0	0	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1		2		3		4		5		6		7		0	

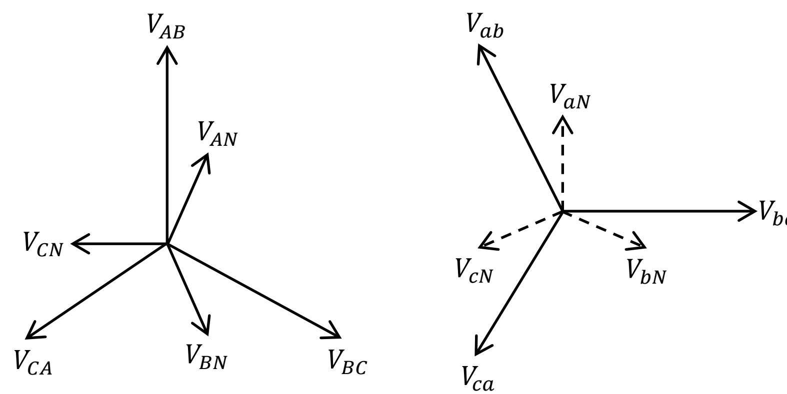
One row and one column of zeros are added to the delta-connected winding to make sure that the primary and second windings have the same size of admittance matrix.

Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer



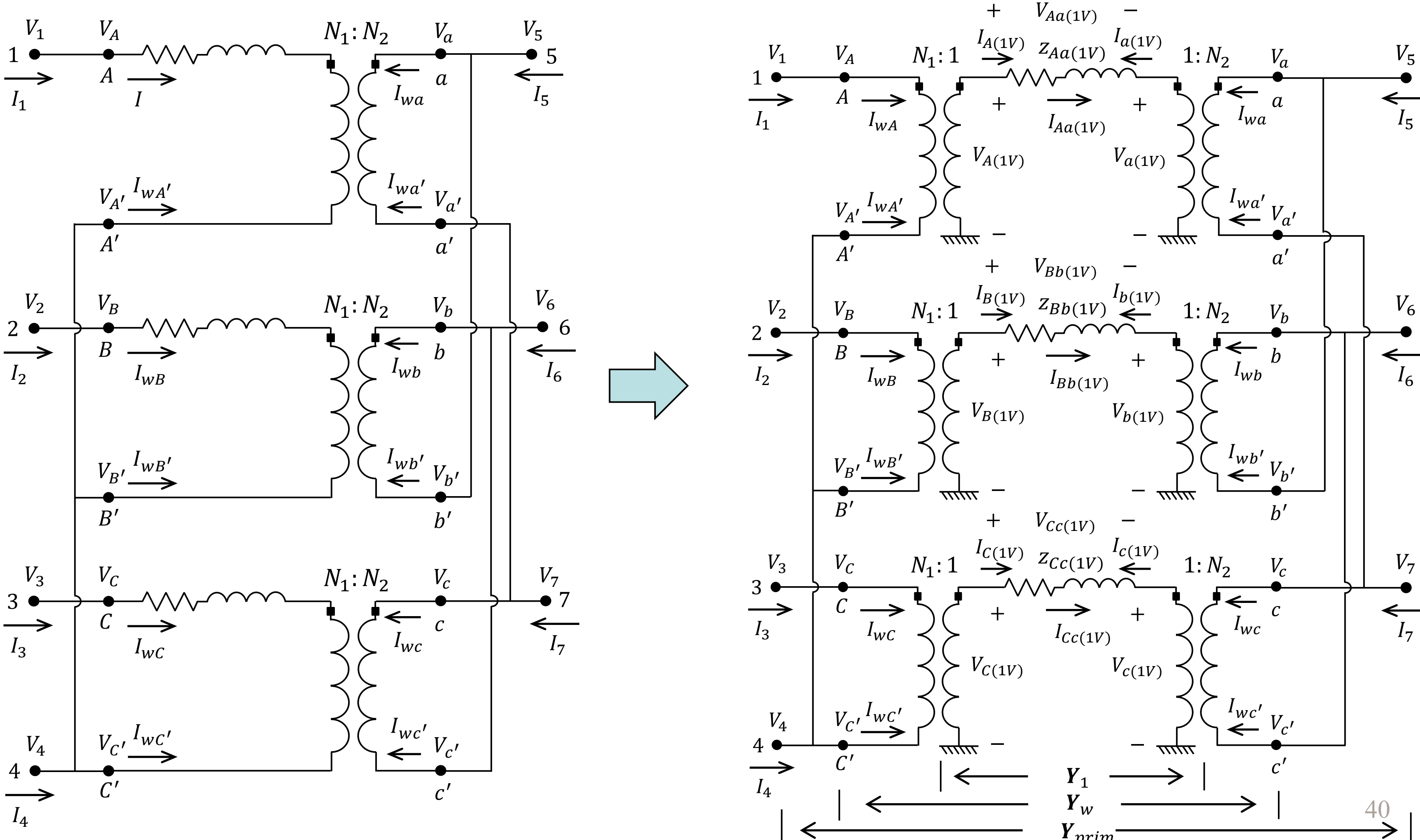
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.



Dyn1

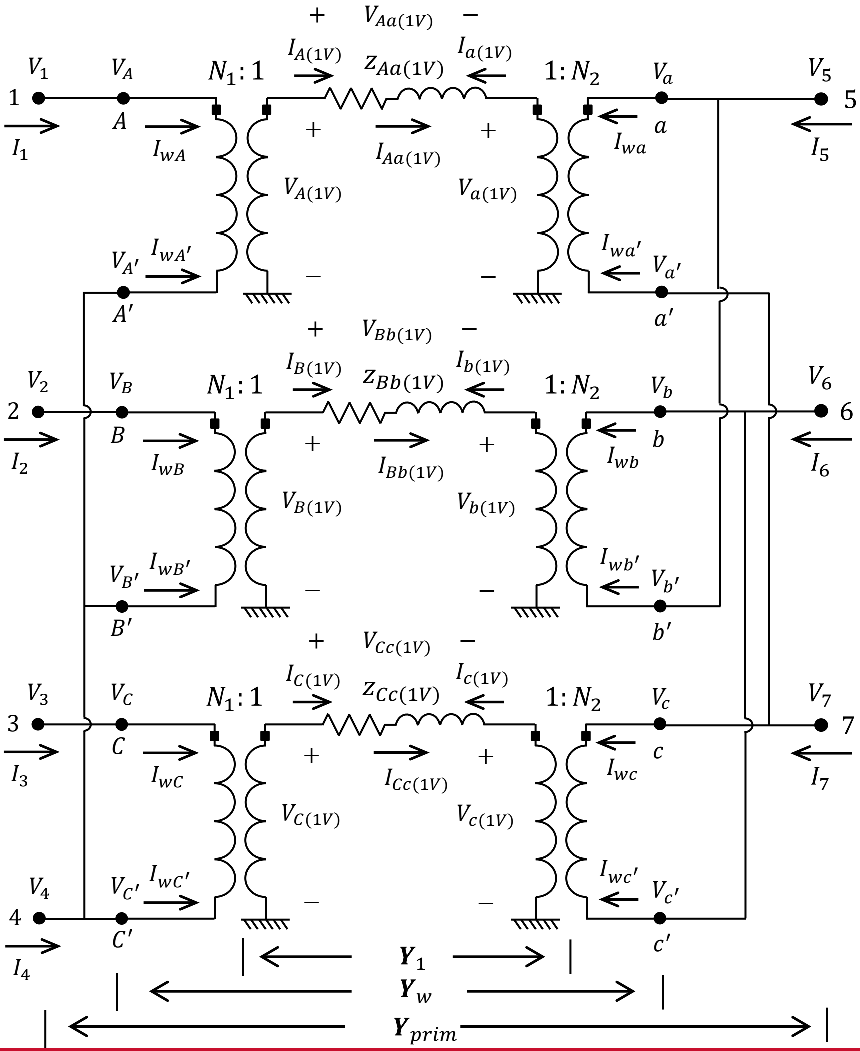
Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer



Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer



Note:

Compared to the 3-phase ungrounded wye-delta step-down transformer, the equation, $\boldsymbol{Y}_{prim} = \boldsymbol{ANB}y_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\,\boldsymbol{A}^{T}$, also applies to a 3-phase ungrounded wye-delta step-up transformer. The only difference exists in the formation of matrix $\boldsymbol{A}$ due to a different connection of delta winding, and the values of N_1 and N_2. Specifically, according to the figure on the left, we have

$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wA'} + I_{wB'} + I_{wC'} \\ I_5 = I_{wa} + I_{wb'} \\ I_6 = I_{wb} + I_{wc'} \\ I_7 = I_{wc} + I_{wa'} \end{cases} \Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0&0&1&0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{A} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0&0&1&0 \end{bmatrix}$$

Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3.4  conn=wye    kV=4.16   kva=500 %r=1
~ wdg=2 bus=M.1.2.3    conn=delta  kV=115    kva=500 %r=1
```

Transformer name

Number of phases

Number of windings

Percent reactance high-to-low

Specify which winding will be edited

Winding connection

Rated voltage

Base kVA rating

Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3*(0.02+j0.05)} = 1149425.2873 - j2873563.2184$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

5. Three-phase Ungrounded Wye-Delta Step-up Transformer

$$\boldsymbol{Y}_{sc(1V)}=\begin{bmatrix} y_{Aa(1V)} & 0 & 0\\ 0 & y_{Bb(1V)} & 0\\ 0 & 0 & y_{Cc(1V)}\end{bmatrix}=\begin{bmatrix} 1149425.2873-j2873563.2184 & 0 & 0\\ 0 & 1149425.2873-j2873563.2184 & 0\\ 0 & 0 & 1149425.2873-j2873563.2184\end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim}=\boldsymbol{ANBY}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\,\boldsymbol{A}^{T}=$

$$\begin{bmatrix}
0.19925780-j0.49814451 & 0+j0 & 0+j0 & -0.19925780+j0.49814451 & -0.00416150+\mathrm{j}0.01040375 & 0+j0 & 0.00416150-\mathrm{j}0.01040375\\
0+j0 & 0.19925780-j0.49814451 & 0+j0 & -0.19925780+j0.49814451 & 0.00416150-\mathrm{j}0.01040375 & -0.00416150+\mathrm{j}0.01040375 & 0+j0\\
0+j0 & 0+j0 & 0.19925780-j0.49814451 & -0.19925780+j0.49814451 & 0+j0 & 0.00416150-\mathrm{j}0.01040375 & -0.00416150+\mathrm{j}0.01040375\\
-0.19925780+j0.49814451 & -0.19925780+j0.49814451 & -0.19925780+j0.49814451 & 0.59777341-j1.49443353 & 0+j0 & 0+j0 & 0+j0\\
-0.00416150+\mathrm{j}0.01040375 & 0.00416150-\mathrm{j}0.01040375 & 0+j0 & 0+j0 & 0.00017383-j0.00043457 & -8.69130652e-05+0.00021728 & -8.69130652e-05+0.00021728\\
0+j0 & -0.00416150+\mathrm{j}0.01040375 & 0.00416150-\mathrm{j}0.01040375 & 0+j0 & -8.69130652e-05+0.00021728 & 0.00017383-j0.00043457 & -8.69130652e-05+0.00021728\\
0.00416150-\mathrm{j}0.01040375 & 0+j0 & -0.00416150+\mathrm{j}0.01040375 & 0+j0 & -8.69130652e-05+0.00021728 & -8.69130652e-05+0.00021728 & 0.00017383-j0.00043457
\end{bmatrix}$$

Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

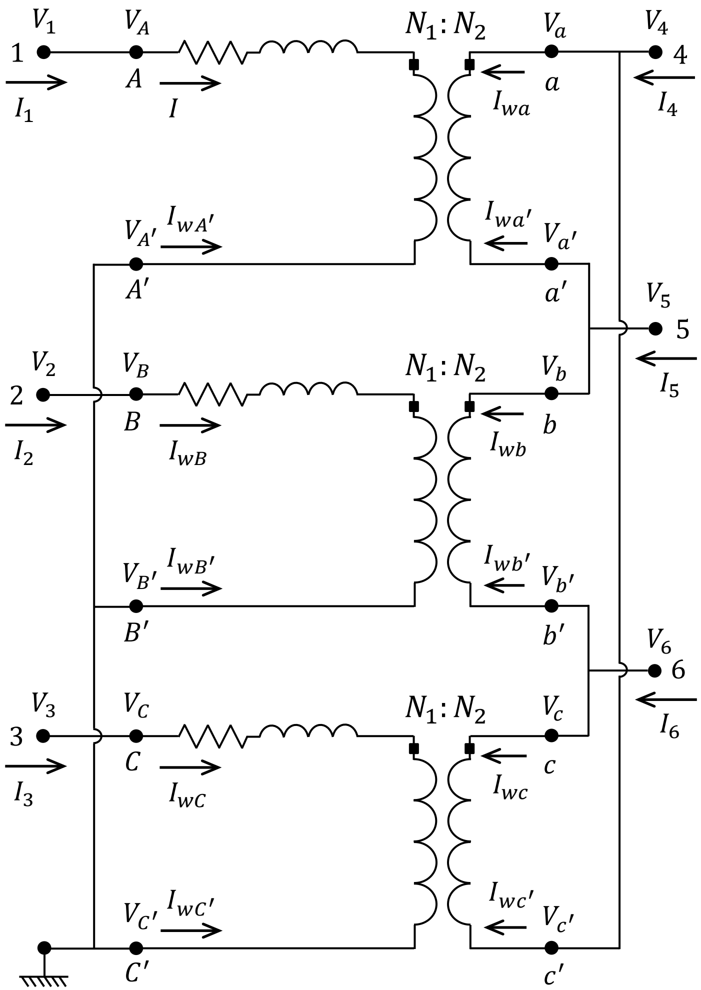
G B

1	0.199258	-0.49814	0	0	0	0	-0.19926	0.498145	-0.00416	0.010404	0	0	0.004162	-0.0104	0	0
2	0	0	0.199258	-0.49814	0	0	-0.19926	0.498145	0.004162	-0.0104	-0.00416	0.010404	0	0	0	0
3	0	0	0	0	0.199258	-0.49814	-0.19926	0.498145	0	0	0.004162	-0.0104	-0.00416	0.010404	0	0
4	-0.19926	0.498145	-0.19926	0.498145	-0.19926	0.498145	0.597773	-1.49443	0	0	0	0	0	0	0	0
5	-0.00416	0.010404	0.004162	-0.0104	0	0	0	0	0.000174	-0.00043	-8.69E-05	0.000217	-8.69E-05	0.000217	0	0
6	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0	-8.69E-05	0.000217	0.000174	-0.00043	-8.69E-05	0.000217	0	0
7	0.004162	-0.0104	0	0	-0.00416	0.010404	0	0	-8.69E-05	0.000217	-8.69E-05	0.000217	0.000174	-0.00043	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1		2		3		4		5		6		7		0	

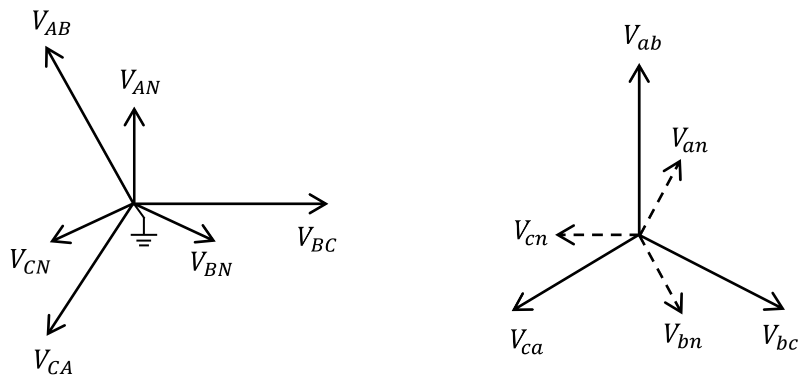
One row and one column of zeros are added to the delta-connected winding to make sure that the primary and second windings have the same size of admittance matrix.

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



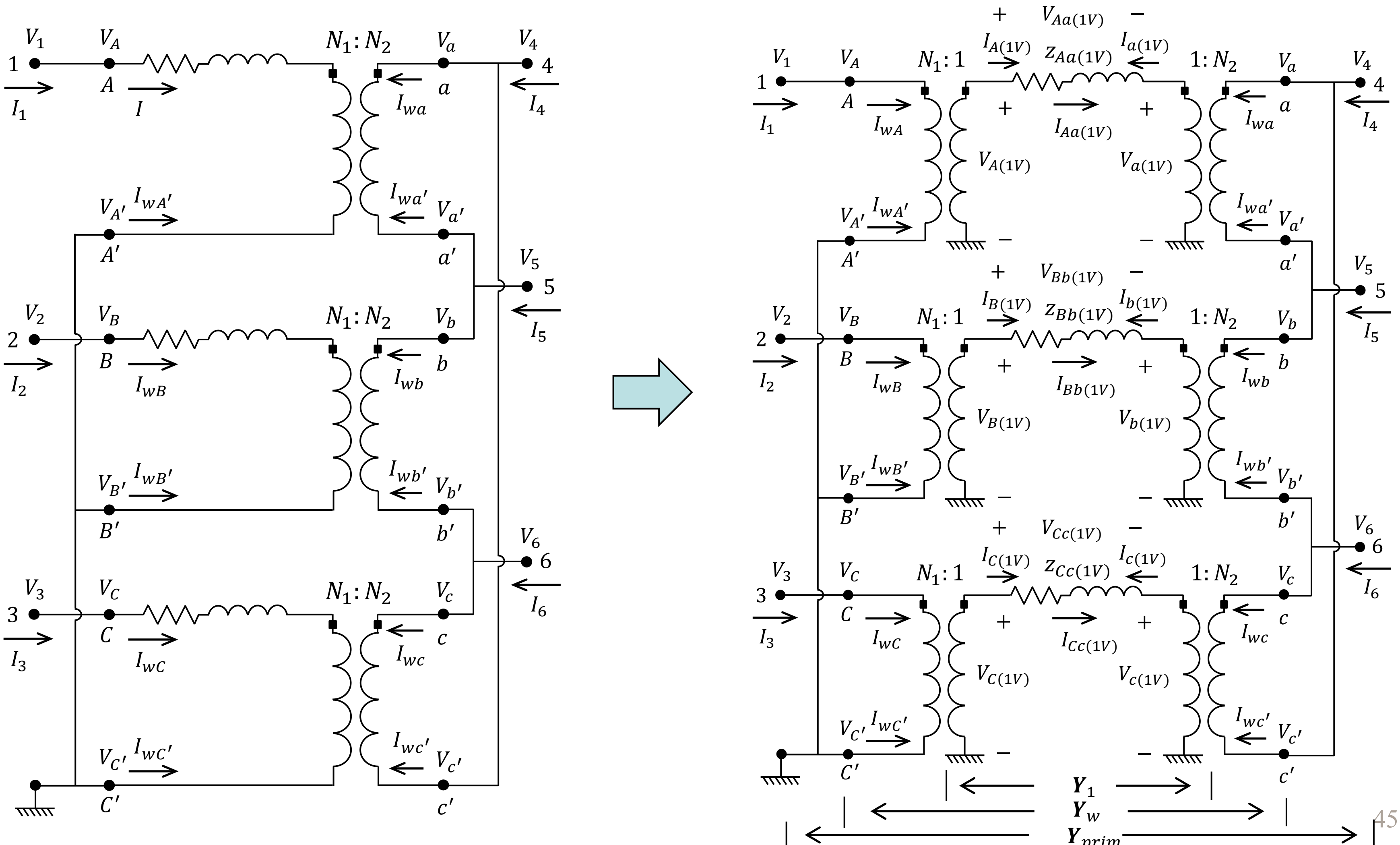
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.



YNd1

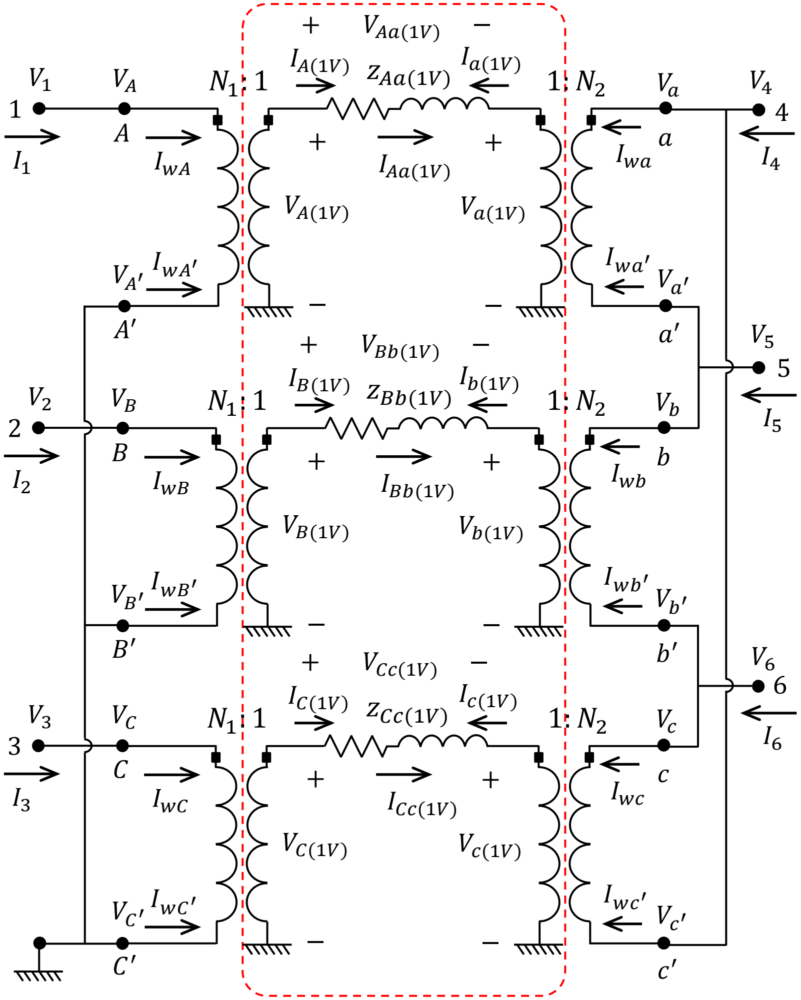
Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.

① $\begin{cases} I_{A(1V)} = I_{Aa(1V)} \\ I_{a(1V)} = -I_{Aa(1V)} \\ I_{B(1V)} = I_{Bb(1V)} \\ I_{b(1V)} = -I_{Bb(1V)} \\ I_{C(1V)} = I_{Cc(1V)} \\ I_{c(1V)} = -I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$

$\Rightarrow \boldsymbol{I}_1 = \boldsymbol{B}\boldsymbol{I}_{sc(1V)}$, where, $\boldsymbol{I}_1 = \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \boldsymbol{I}_{sc(1V)} = \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$

② $\begin{cases} V_{Aa(1V)} = V_{A(1V)} - V_{a(1V)} \\ V_{Bb(1V)} = V_{B(1V)} - V_{b(1V)} \\ V_{Cc(1V)} = V_{C(1V)} - V_{c(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix}$

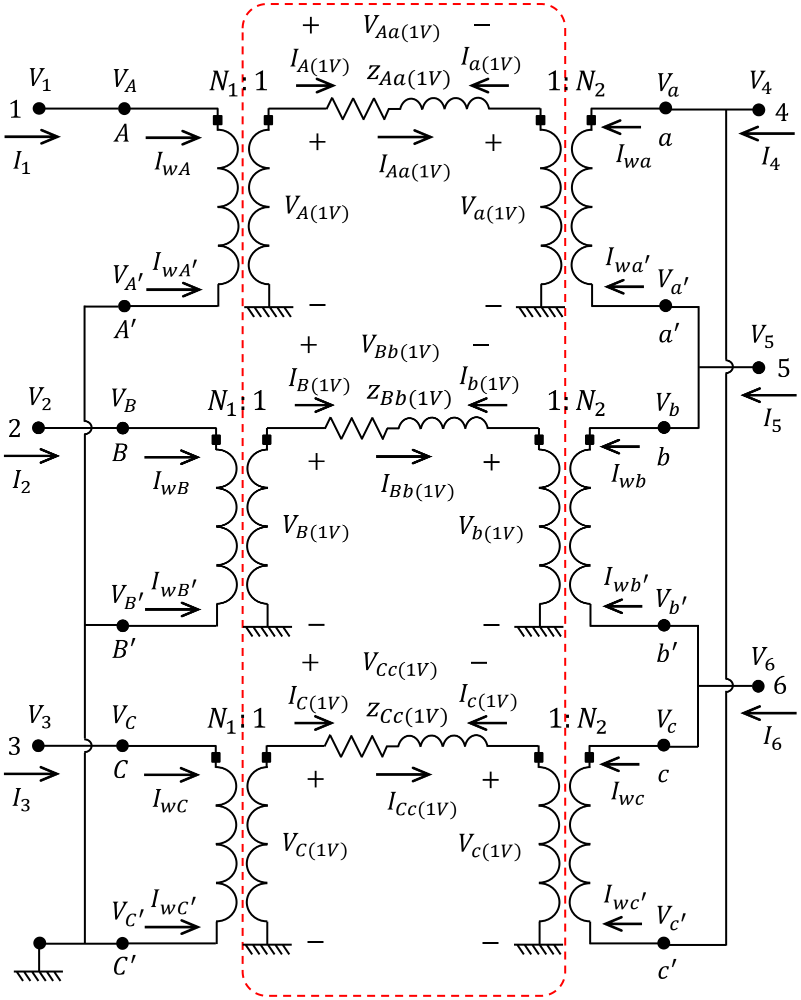
$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{B}^T\boldsymbol{V}_1$, where, $\boldsymbol{V}_{sc(1V)} = \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix}, \boldsymbol{V}_1 = \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ \dot{V}_{c(1V)} \end{bmatrix}$

③ $\begin{cases} V_{Aa(1V)} = z_{Aa(1V)} I_{Aa(1V)} \\ V_{Bb(1V)} = z_{Bb(1V)} I_{Bb(1V)} \\ V_{Cc(1V)} = z_{Cc(1V)} I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} z_{Aa(1V)} & 0 & 0 \\ 0 & z_{Bb(1V)} & 0 \\ 0 & 0 & z_{Cc(1V)} \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$

$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{Z}_{sc(1V)}\boldsymbol{I}_{sc(1V)} \Rightarrow \boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} = \boldsymbol{I}_{sc(1V)}$, where, $\boldsymbol{Y}_{sc(1V)} = (\boldsymbol{Z}_{sc(1V)})^{-1}$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Let $I_1 = Y_1 V_1$, where, Y_1 is the admittance matrix on a one turn or one voltage base. What is Y_1?

$$\left.\begin{aligned} I_1 &= Y_1 V_1 \\ I_1 &= B I_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} B I_{sc(1V)} &= Y_1 V_1 \\ V_{sc(1V)} &= B^T V_1 \end{aligned}\right\}$$

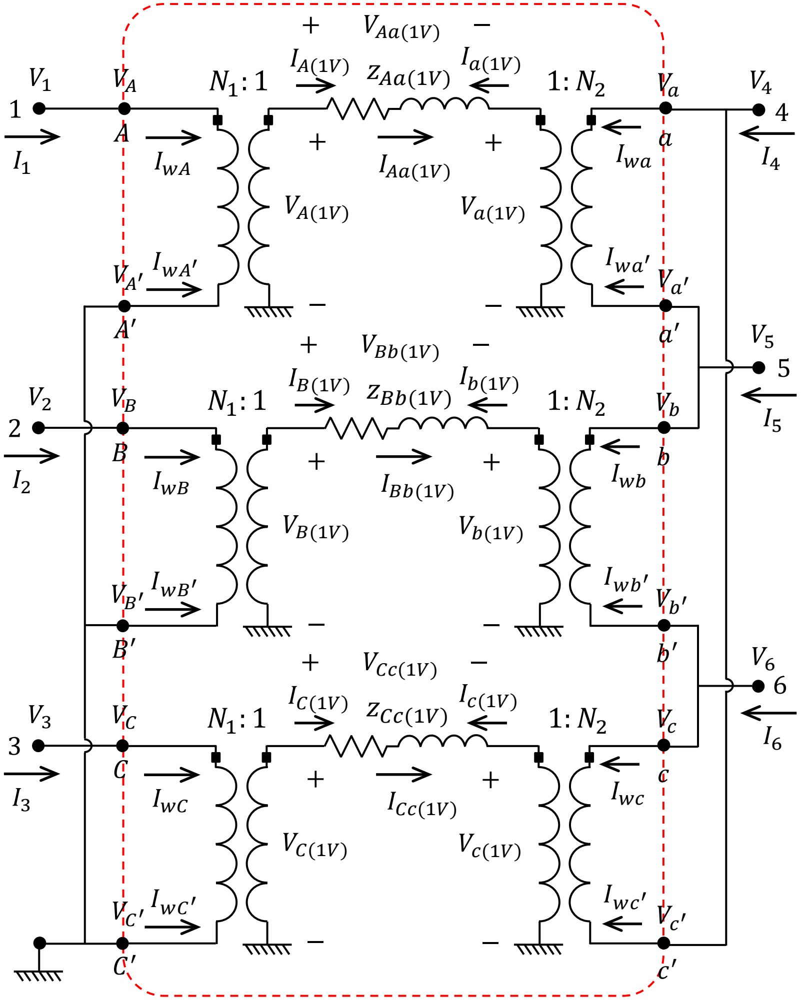
$$\Rightarrow \left.\begin{aligned} B I_{sc(1V)} &= Y_1 \left(B^{\mathrm{T}}\right)^{-1} V_{sc(1V)} \\ I_{sc(1V)} &= Y_{sc(1V)} V_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow B Y_{sc(1V)} V_{sc(1V)} = Y_1 \left(B^{\mathrm{T}}\right)^{-1} V_{sc(1V)}$$

$$\Rightarrow Y_1 = B Y_{sc(1V)} B^{\mathrm{T}}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Step 2: Consider winding turns ratio.

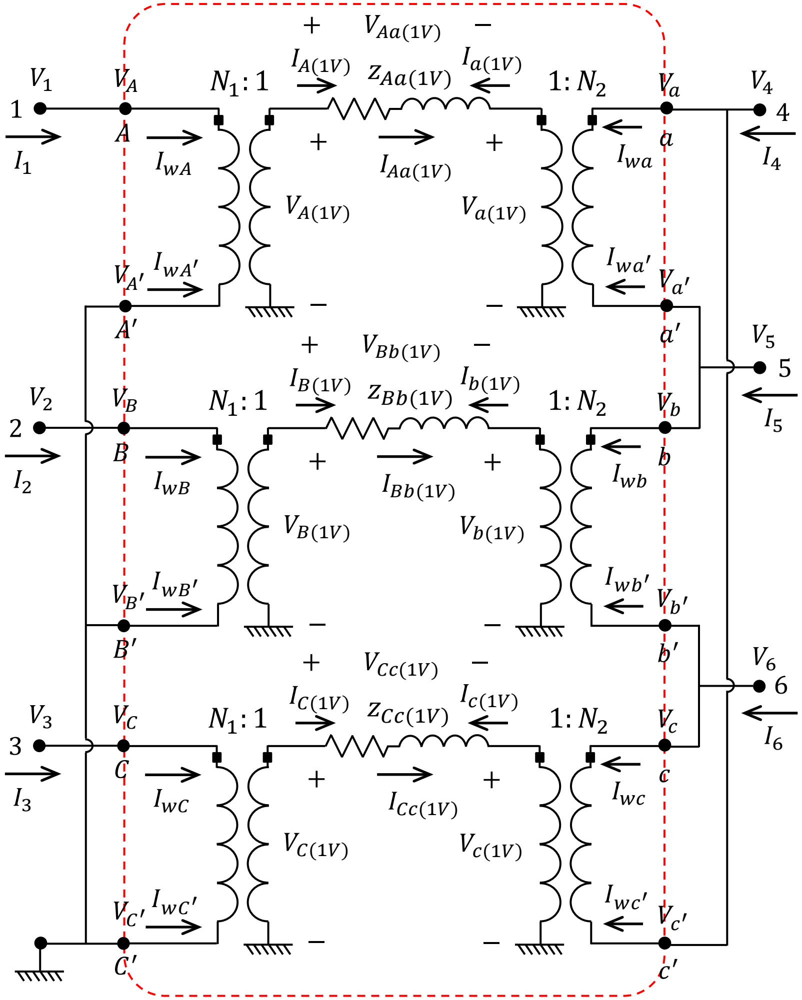
①
$$\begin{cases} I_{wA} = I_{A(1V)}/N_1 \\ I_{wA'} = -I_{A(1V)}/N_1 \\ I_{wa} = I_{a(1V)}/N_2 \\ I_{wa'} = -I_{a(1V)}/N_2 \\ I_{wB} = I_{B(1V)}/N_1 \\ I_{wB'} = -I_{B(1V)}/N_1 \\ I_{wb} = I_{b(1V)}/N_2 \\ I_{wb'} = -I_{b(1V)}/N_2 \\ I_{wC} = I_{C(1V)}/N_1 \\ I_{wC'} = -I_{C(1V)}/N_1 \\ I_{wc} = I_{c(1V)}/N_2 \\ I_{wc'} = -I_{c(1V)}/N_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I_w} = \boldsymbol{N}\boldsymbol{I_1}, \text{ where } \boldsymbol{I_w} = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, \ \boldsymbol{N} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



(2)

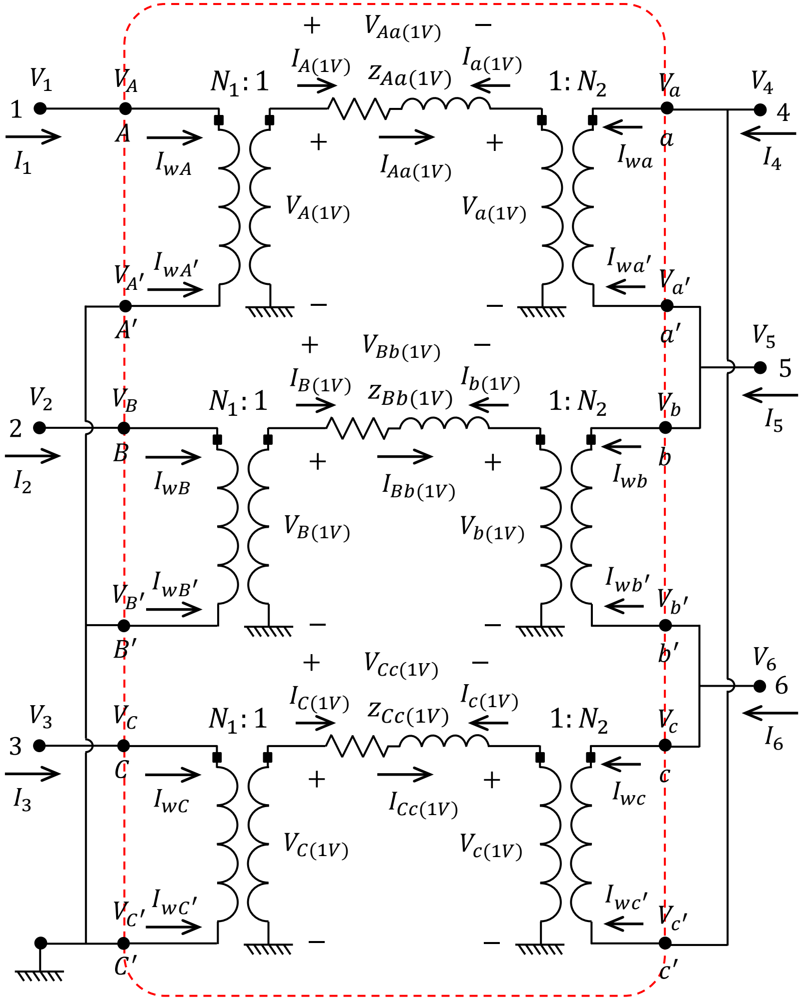
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_1 = \boldsymbol{N}^T \boldsymbol{V}_w, \text{ where, } \boldsymbol{v}_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Next, let $\boldsymbol{I}_w = \boldsymbol{Y}_w \boldsymbol{V}_w$. How to obtain $\boldsymbol{Y}_w$?

$$\left.\begin{aligned} \boldsymbol{I}_w &= \boldsymbol{Y}_w \boldsymbol{V}_w \\ \boldsymbol{I}_w &= \boldsymbol{N}\boldsymbol{I}_1 \\ \boldsymbol{V}_1 &= \boldsymbol{N}^T \boldsymbol{V}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{N}\boldsymbol{I}_1 &= \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \end{aligned}\right\}$$

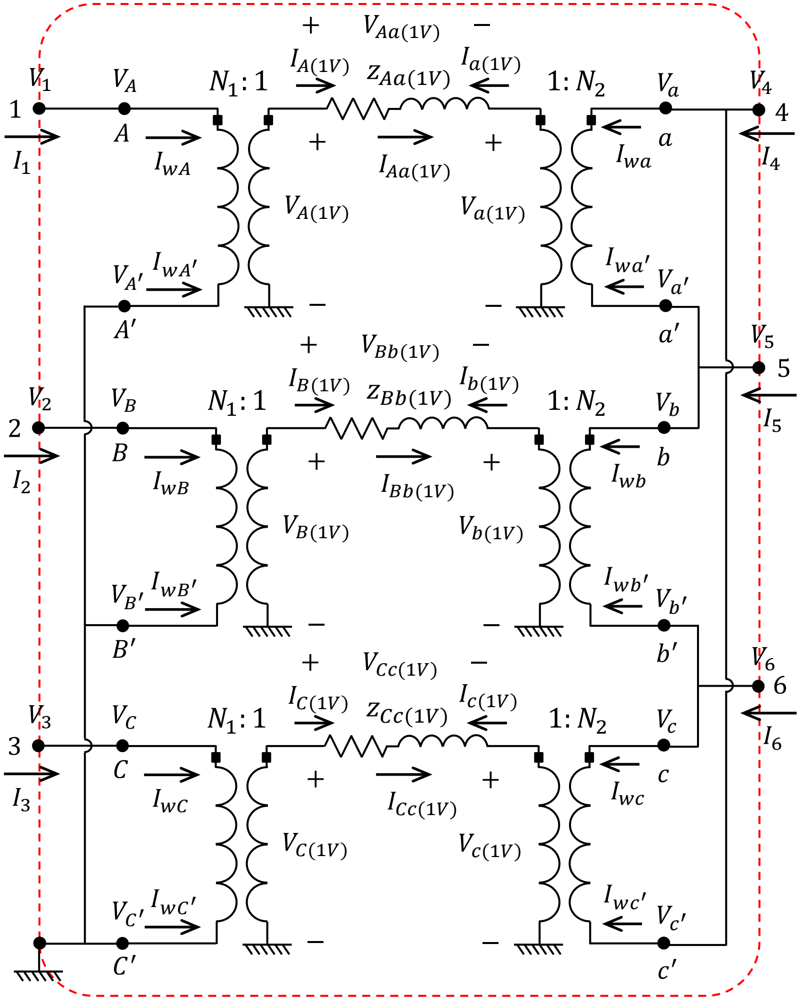
$$\Rightarrow \boldsymbol{N}\boldsymbol{Y}_1 \boldsymbol{V}_1 = \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{Y}_1 \boldsymbol{N}^T \\ \boldsymbol{Y}_1 &= \boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_w = \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \boldsymbol{N}^T$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Step 3: Consider winding connections

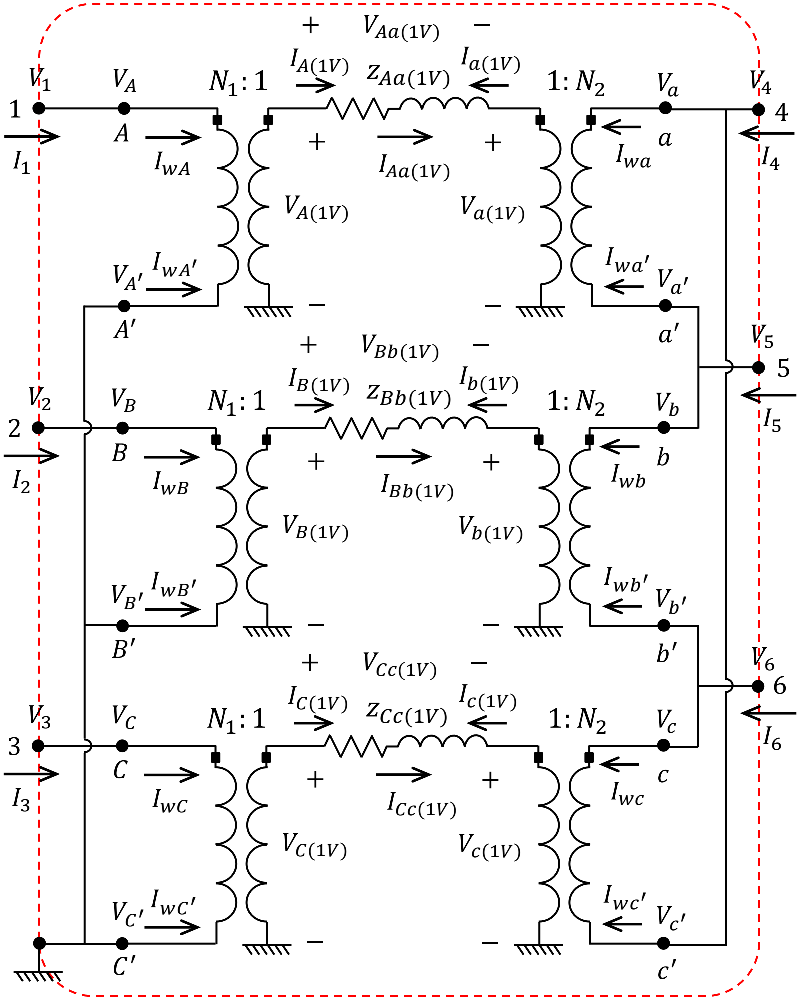
①
$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wa} + I_{wc'} \\ I_5 = I_{wa'} + I_{wb} \\ I_6 = I_{wb'} + I_{wc} \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_w, \text{ where, } \boldsymbol{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



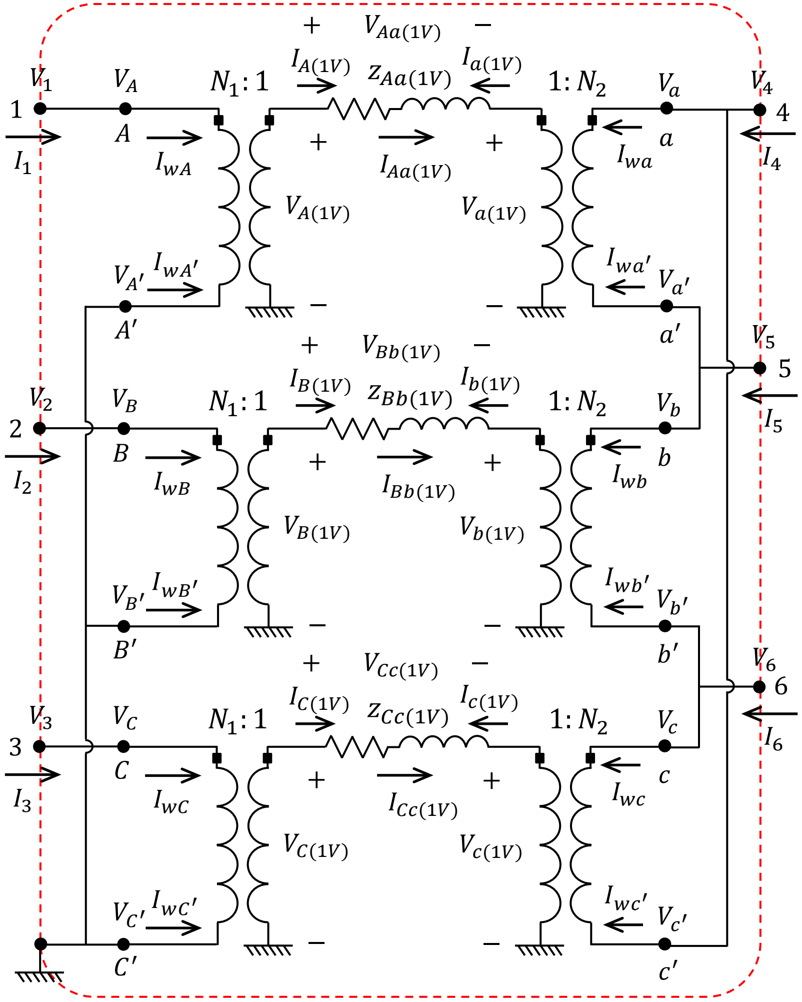
(2)

$$
\begin{cases}
V_A = V_1 \\
V_{A'} = 0 \\
V_a = V_4 \\
V_{a'} = V_5 \\
V_B = V_2 \\
V_{B'} = 0 \\
V_b = V_5 \\
V_{b'} = V_6 \\
V_C = V_3 \\
V_{C'} = 0 \\
V_c = V_6 \\
V_{c'} = V_4
\end{cases}
\Rightarrow
\begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}
$$

$\Rightarrow$ $\boldsymbol{V}_w = \boldsymbol{A}^T \boldsymbol{V}_{prim}$, where, $\boldsymbol{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$.

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer



Let $\boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim}$. How to obtain $\boldsymbol{Y}_{prim}$?

$$\left.\begin{aligned} \boldsymbol{I}_{prim} &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim} \\ \boldsymbol{I}_{prim} &= \boldsymbol{A}\boldsymbol{I}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{I}_w &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim} \\ \boldsymbol{V}_w &= \boldsymbol{A}^T\,\boldsymbol{V}_{prim} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{I}_w &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^T)^{-1}\boldsymbol{V}_w \\ \boldsymbol{I}_w &= \boldsymbol{Y}_w\,\boldsymbol{V}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{Y}_w\,\boldsymbol{V}_w &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^T)^{-1}\boldsymbol{V}_w \\ \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_{prim} = \boldsymbol{A}\boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T\,\boldsymbol{A}^T$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3.0  conn=wye   kV=115  kva=500 %r=1
~ wdg=2 bus=M.1.2.3    conn=delta kV=4.16 kva=500 %r=1
```

- Transformer name
- Number of phases
- Number of windings
- Percent reactance high-to-low
- Specify which winding will be edited
- Winding connection
- Rated voltage
- Base kVA rating
- Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184\ S$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

6. Three-phase Grounded Wye-Delta Step-down Transformer

$$\boldsymbol{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANBY}_{sc(1V)}\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\ \boldsymbol{A}^{T}$ =

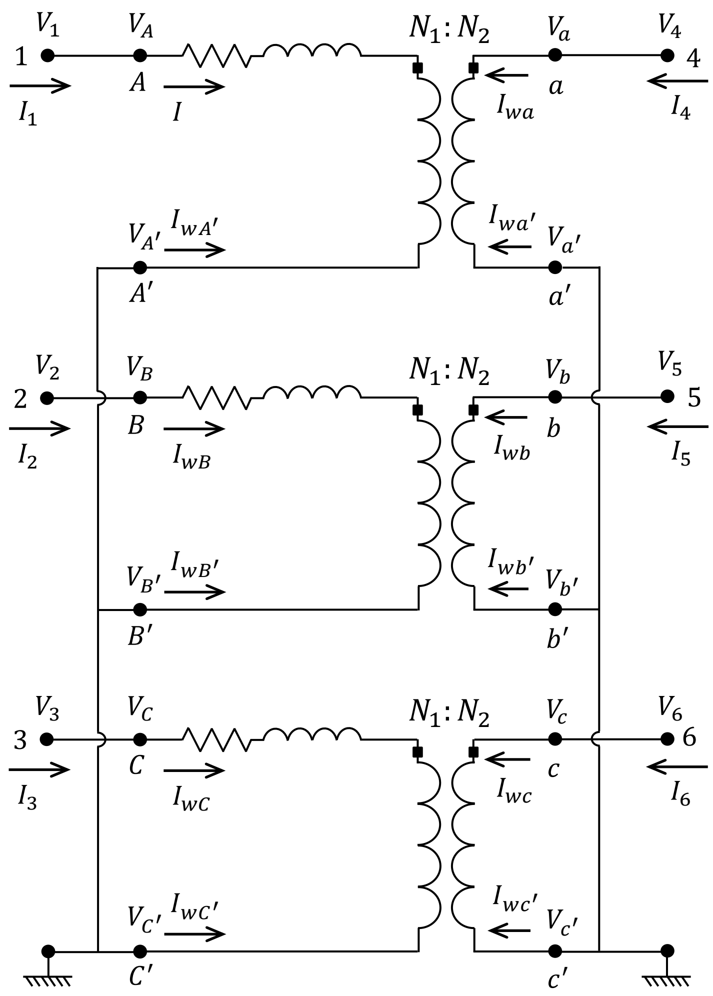
$$\begin{bmatrix} 0.000260739 - j0.00065184 & 0+j0 & 0+j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 & 0+j0 \\ 0+j0 & 0.000260739 - j0.00065184 & 0+j0 & 0+j0 & -0.00416150 + j0.01040375 & 0.00416150 - j0.01040375 \\ 0+j0 & 0+j0 & 0.000260739 - j0.00065184 & 0.00416150 - j0.01040375 & 0+j0 & -0.00416150 + j0.01040375 \\ -0.00416150 + j0.01040375 & 0+j0 & 0.00416150 - j0.01040375 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 \\ 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & 0+j0 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 & -0.06641926 + j0.16604817 \\ 0+j0 & 0.00416150 - j0.01040375 & -0.00416150 + j0.01040375 & -0.06641926 + j0.16604817 & -0.06641926 + j0.16604817 & 0.13283853 - j0.33209634 \end{bmatrix}$$

Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

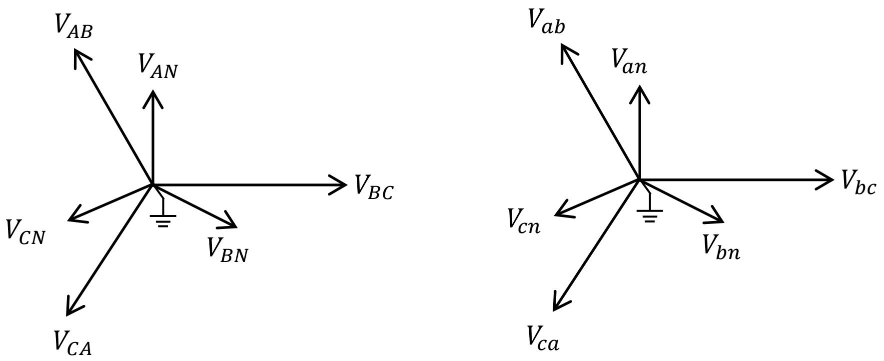
					G	B						
1	0.000261	-0.00065	0	0	0	0	-0.00416	0.010404	0.004162	-0.0104	0	0
2	0	0	0.000261	-0.00065	0	0	0	0	-0.00416	0.010404	0.004162	-0.0104
3	0	0	0	0	0.000261	-0.00065	0.004162	-0.0104	0	0	-0.00416	0.010404
4	-0.00416	0.010404	0	0	0.004162	-0.0104	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048
5	0.004162	-0.0104	-0.00416	0.010404	0	0	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048
6	0	0	0.004162	-0.0104	-0.00416	0.010404	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321
	1		2		3		4		5		6	

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



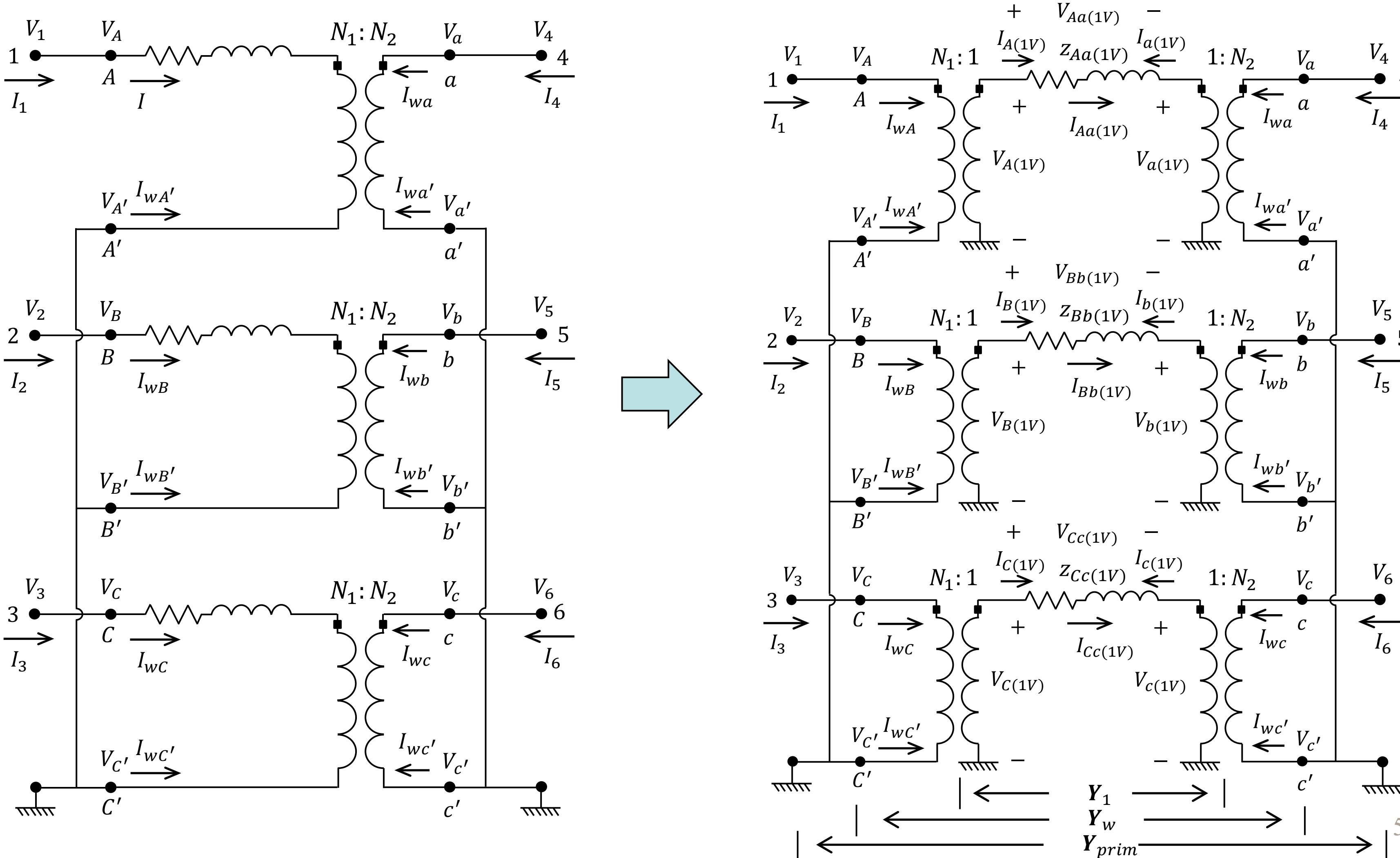
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.



YNyn0

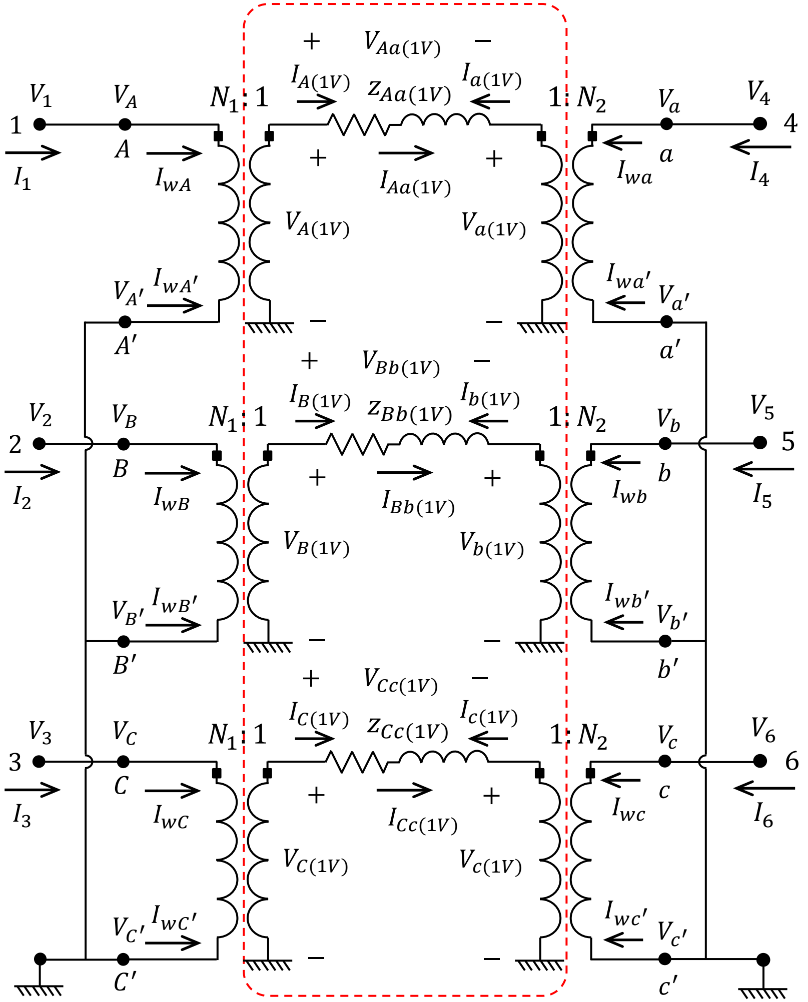
Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.

(1)
$$\begin{cases} I_{A(1V)} = I_{Aa(1V)} \\ I_{a(1V)} = -I_{Aa(1V)} \\ I_{B(1V)} = I_{Bb(1V)} \\ I_{b(1V)} = -I_{Bb(1V)} \\ I_{C(1V)} = I_{Cc(1V)} \\ I_{c(1V)} = -I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_1 = \boldsymbol{B}\boldsymbol{I}_{sc(1V)}, \text{ where, } \boldsymbol{I}_1 = \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \boldsymbol{I}_{sc(1V)} = \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

(2)
$$\begin{cases} V_{Aa(1V)} = V_{A(1V)} - V_{a(1V)} \\ V_{Bb(1V)} = V_{B(1V)} - V_{b(1V)} \\ V_{Cc(1V)} = V_{C(1V)} - V_{c(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix}$$

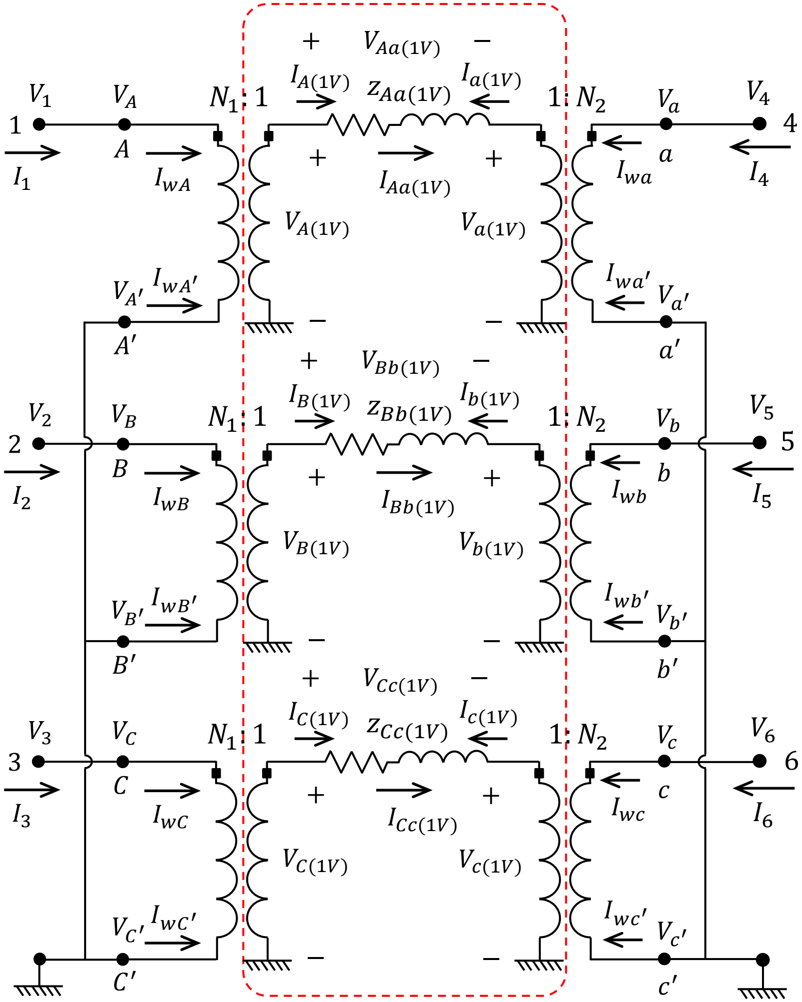
$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{B}^T\boldsymbol{V}_1, \text{ where, } \boldsymbol{V}_{sc(1V)} = \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix}, \boldsymbol{V}_1 = \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ \dot{V}_{c(1V)} \end{bmatrix}$$

(3)
$$\begin{cases} V_{Aa(1V)} = z_{Aa(1V)} I_{Aa(1V)} \\ V_{Bb(1V)} = z_{Bb(1V)} I_{Bb(1V)} \\ V_{Cc(1V)} = z_{Cc(1V)} I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} z_{Aa(1V)} & 0 & 0 \\ 0 & z_{Bb(1V)} & 0 \\ 0 & 0 & z_{Cc(1V)} \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{Z}_{sc(1V)}\boldsymbol{I}_{sc(1V)} \Rightarrow \boldsymbol{Y}_{sc(1V)}\boldsymbol{V}_{sc(1V)} = \boldsymbol{I}_{sc(1V)}, \quad where, \boldsymbol{Y}_{sc(1V)} = (\boldsymbol{Z}_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Let $\boldsymbol{I}_1 = \boldsymbol{Y}_1 \boldsymbol{V}_1$, where, $\boldsymbol{Y}_1$ is the admittance matrix on a one turn or one voltage base. What is $\boldsymbol{Y}_1$?

$$\left.\begin{aligned} \boldsymbol{I}_1 &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{B} \boldsymbol{I}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{B} \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \\ \boldsymbol{V}_{sc(1V)} &= \boldsymbol{B}^T \boldsymbol{V}_1 \end{aligned}\right\}$$

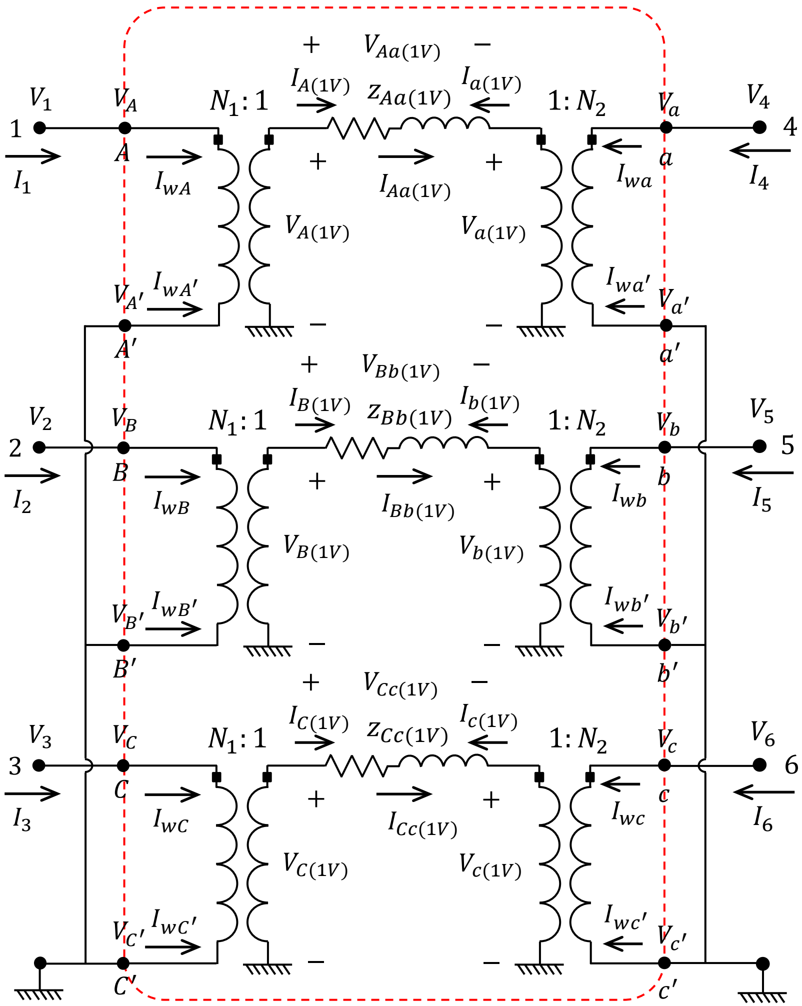
$$\Rightarrow \left.\begin{aligned} \boldsymbol{B} \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1 \left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1} \boldsymbol{V}_{sc(1V)} \\ \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_{sc(1V)} \boldsymbol{V}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{B} \boldsymbol{Y}_{sc(1V)} \boldsymbol{V}_{sc(1V)} = \boldsymbol{Y}_1 \left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1} \boldsymbol{V}_{sc(1V)}$$

$$\Rightarrow \boldsymbol{Y}_1 = \boldsymbol{B} \boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



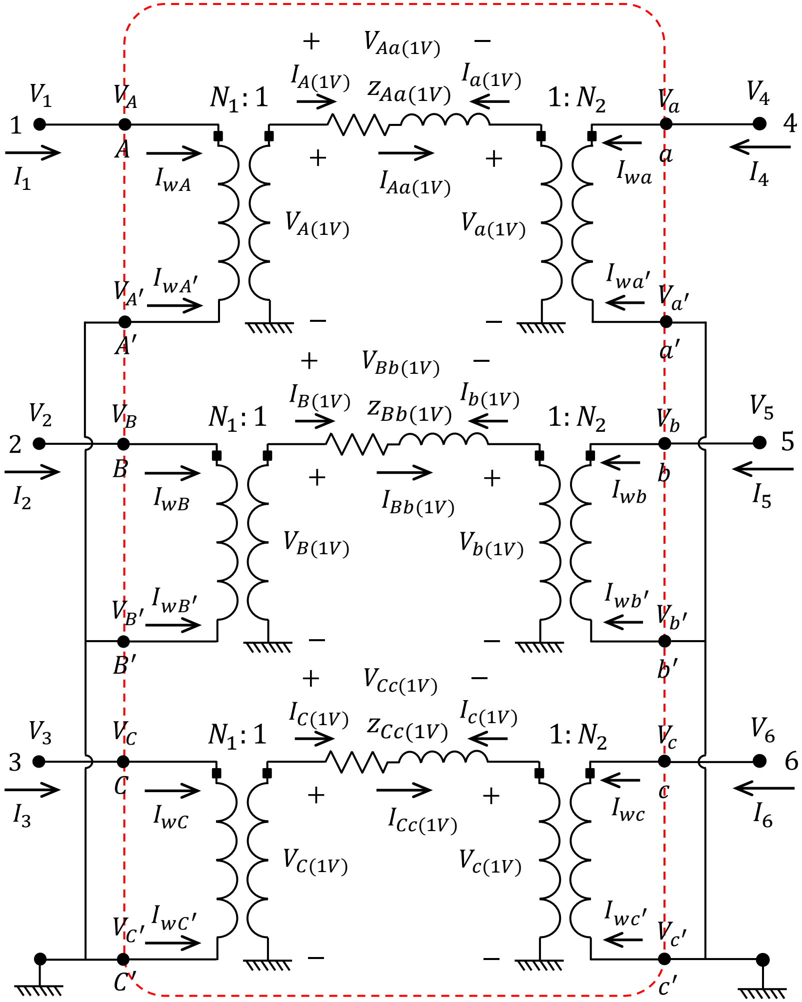
Step 2: Consider winding turns ratio.

(1)
$$
\begin{cases}
I_{wA} = I_{A(1V)}/N_1 \\
I_{wA'} = -I_{A(1V)}/N_1 \\
I_{wa} = I_{a(1V)}/N_2 \\
I_{wa'} = -I_{a(1V)}/N_2 \\
I_{wB} = I_{B(1V)}/N_1 \\
I_{wB'} = -I_{B(1V)}/N_1 \\
I_{wb} = I_{b(1V)}/N_2 \\
I_{wb'} = -I_{b(1V)}/N_2 \\
I_{wC} = I_{C(1V)}/N_1 \\
I_{wC'} = -I_{C(1V)}/N_1 \\
I_{wc} = I_{c(1V)}/N_2 \\
I_{wc'} = -I_{c(1V)}/N_2
\end{cases}
\Rightarrow
\begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}
=
\begin{bmatrix}
\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2}
\end{bmatrix}
\begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}
$$

$$
\Rightarrow \boldsymbol{I}_w = \boldsymbol{N}\boldsymbol{I}_1, \text{ where } \boldsymbol{I}_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}, \ \boldsymbol{N} =
\begin{bmatrix}
\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2}
\end{bmatrix}
$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



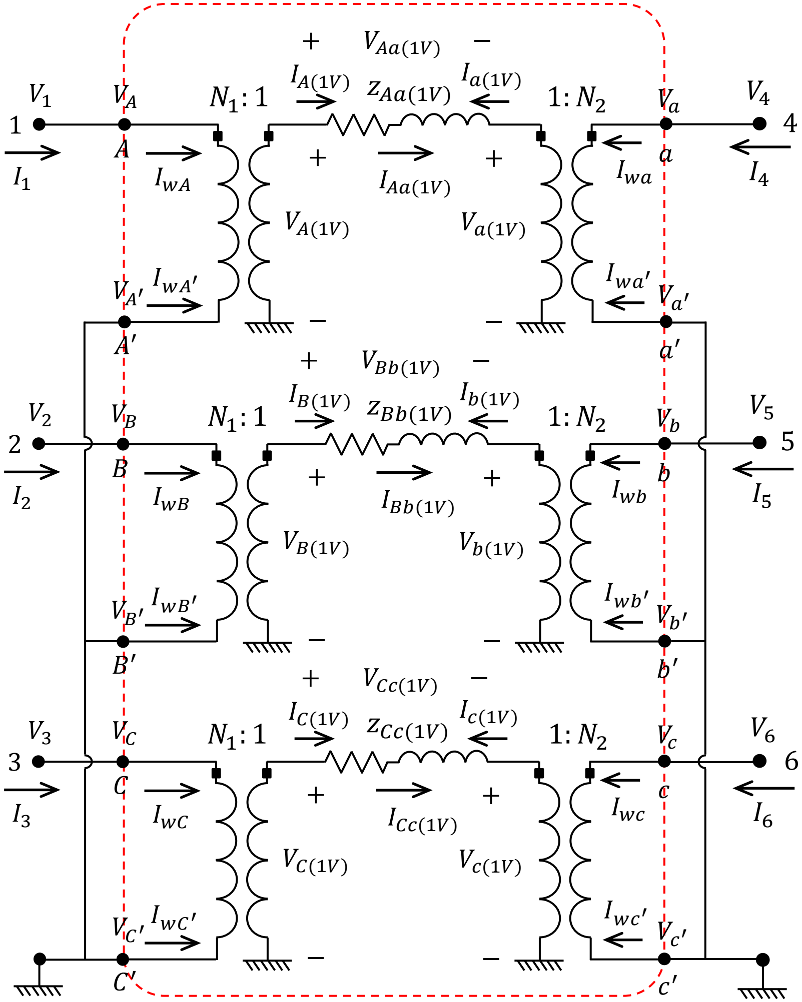
②
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ \dot{V}_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_1 = \boldsymbol{N}^T \boldsymbol{V}_w, \text{ where, } \boldsymbol{v}_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Next, let $\boldsymbol{I}_w = \boldsymbol{Y}_w \boldsymbol{V}_w$. How to obtain $\boldsymbol{Y}_w$?

$$\left.\begin{aligned} \boldsymbol{I}_w &= \boldsymbol{Y}_w \boldsymbol{V}_w \\ \boldsymbol{I}_w &= \boldsymbol{N}\boldsymbol{I}_1 \\ \boldsymbol{V}_1 &= \boldsymbol{N}^T \boldsymbol{V}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{N}\boldsymbol{I}_1 &= \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \end{aligned}\right\}$$

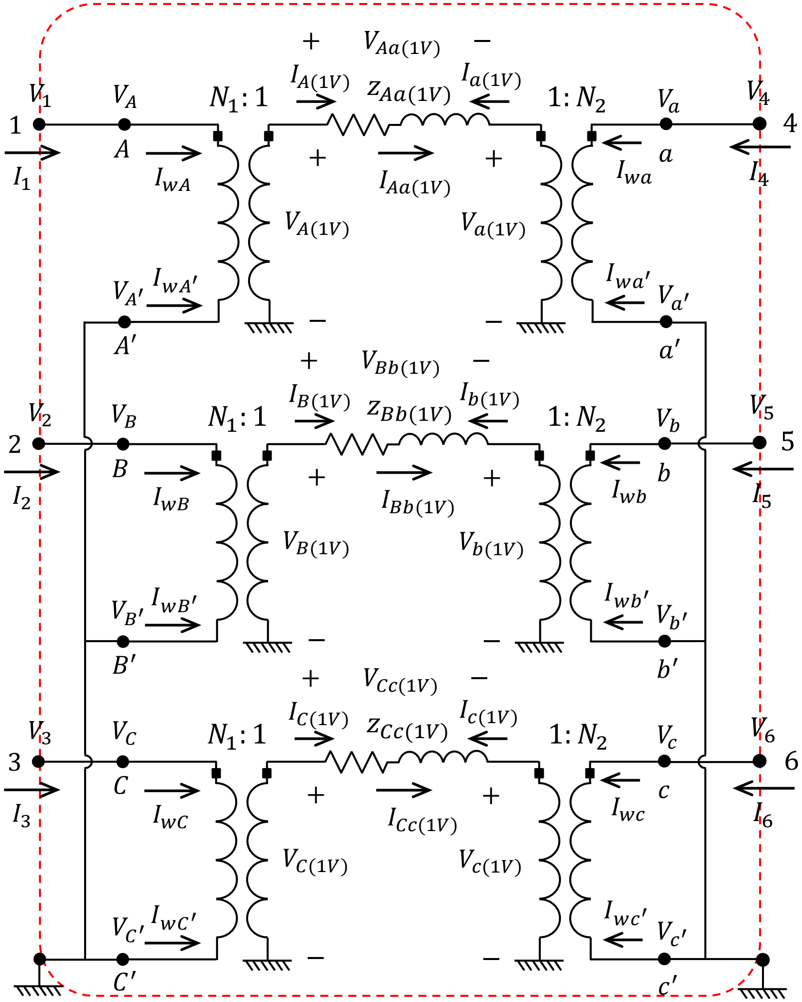
$$\Rightarrow \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{V}_1 = \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{N}^T \\ \boldsymbol{Y}_1 &= \boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_w = \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Step 3: Consider winding connections

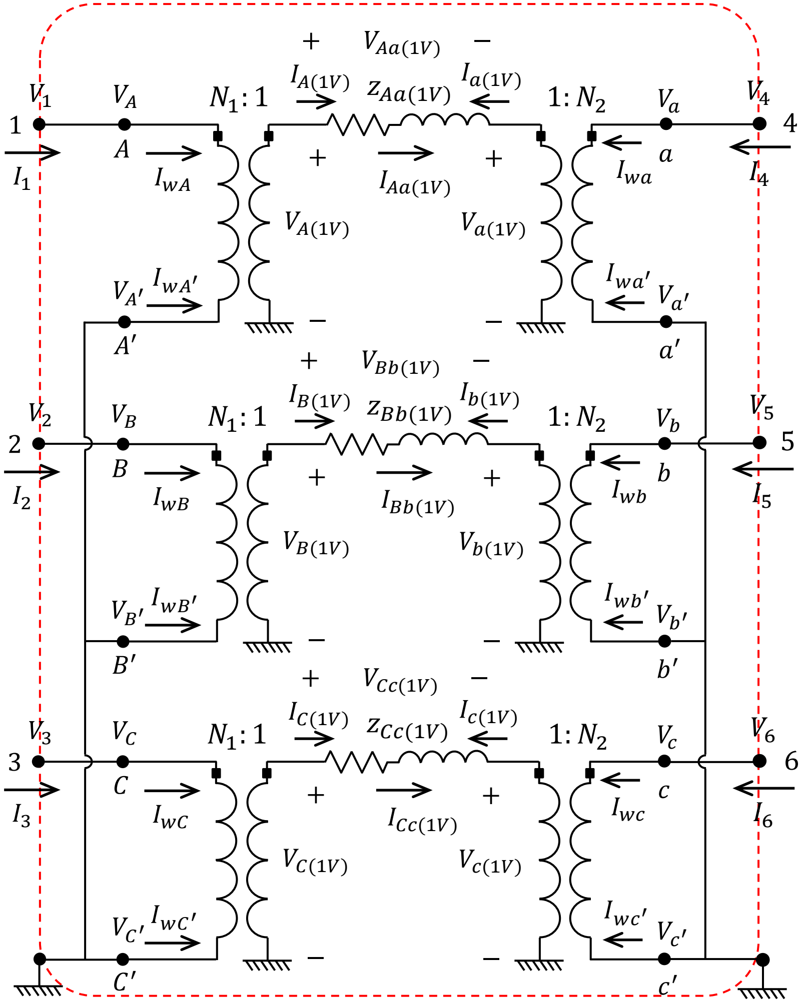
(1)
$$\begin{cases} I_1 = I_{wA} \\ I_2 = I_{wB} \\ I_3 = I_{wC} \\ I_4 = I_{wa} \\ I_5 = I_{wb} \\ I_6 = I_{wc} \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_w, \text{ where, } \boldsymbol{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

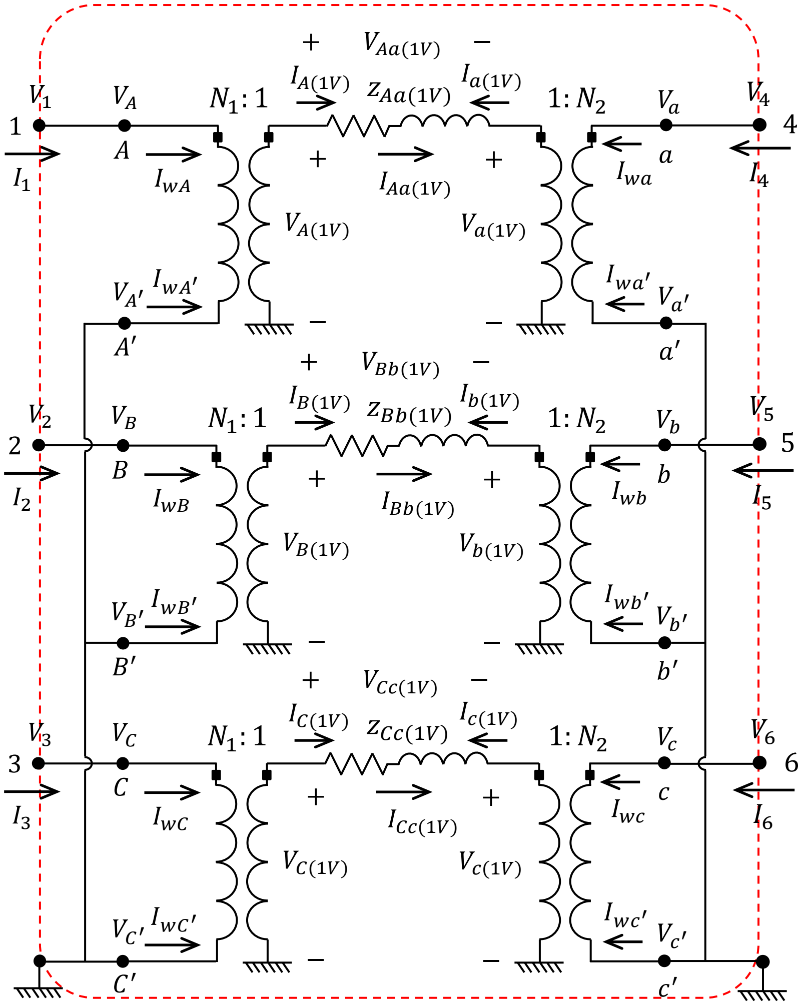


②
$$\begin{cases} V_A = V_1 \\ V_{A'} = 0 \\ V_a = V_4 \\ V_{a'} = 0 \\ V_B = V_2 \\ V_{B'} = 0 \\ V_b = V_5 \\ V_{b'} = 0 \\ V_C = V_3 \\ V_{C'} = 0 \\ V_c = V_6 \\ V_{c'} = 0 \end{cases} \Rightarrow \begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

$\Rightarrow$ $\boldsymbol{V}_w = \boldsymbol{A}^T \boldsymbol{V}_{prim}$, where, $\boldsymbol{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$.

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer



Let $\boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim}$. How to obtain $\boldsymbol{Y}_{prim}$?

$$\left.\begin{aligned} \boldsymbol{I}_{prim} &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim} \\ \boldsymbol{I}_{prim} &= \boldsymbol{A}\boldsymbol{I}_{w} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{I}_{w} &= \boldsymbol{Y}_{prim}\,\boldsymbol{V}_{prim} \\ \boldsymbol{V}_{w} &= \boldsymbol{A}^{T}\,\boldsymbol{V}_{prim} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{I}_{w} &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w} \\ \boldsymbol{I}_{w} &= \boldsymbol{Y}_{w}\,\boldsymbol{V}_{w} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{A}\boldsymbol{Y}_{w}\,\boldsymbol{V}_{w} &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w} \\ \boldsymbol{Y}_{w} &= \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_{prim} = \boldsymbol{A}\boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\,\boldsymbol{A}^{T}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

An example:

Transformer name

Number of phases

Number of windings

Percent reactance high-to-low

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3.0    conn=wye    kV=115    kva=500 %r=1
~ wdg=2 bus=M.1.2.3.0    conn=wye    kV=4.16   kva=500 %r=1
```

Specify which winding will be edited

Winding connection

Rated voltage

Base kVA rating

Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 - j2873563.2184$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000/\sqrt{3}} & -\frac{1}{115000/\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160/\sqrt{3}} & -\frac{1}{4160/\sqrt{3}} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

7. Three-phase Grounded Wye-Grounded Wye Transformer

$$\boldsymbol{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANBY}_{sc(1V)}\,\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\,\boldsymbol{A}^{T} =$

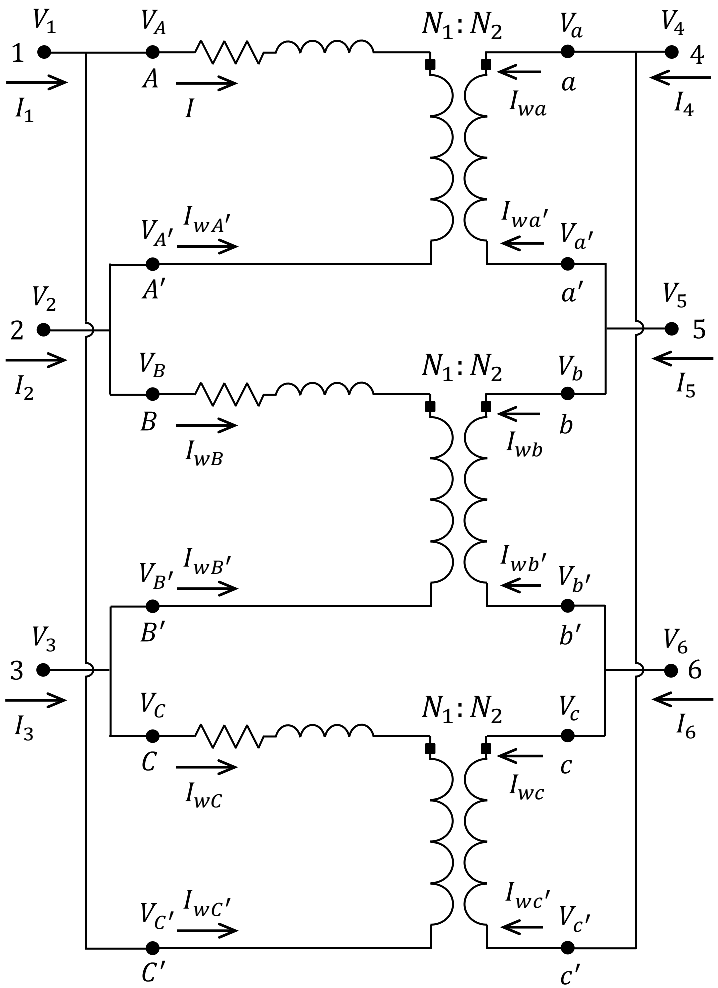
$$\begin{bmatrix} 0.000260739 - j0.00065184 & 0+j0 & 0+j0 & -0.00720793 + j0.01801984 & 0+j0 & 0+j0 \\ 0+j0 & 0.000260739 - j0.00065184 & 0+j0 & 0+j0 & -0.00720793 + j0.01801984 & 0+j0 \\ 0+j0 & 0+j0 & 0.000260739 - j0.00065184 & 0+j0 & 0+j0 & -0.00720793 + j0.01801984 \\ -0.00720793 + j0.01801984 & 0+j0 & 0+j0 & 0.19925780 - j0.49814451 & 0+j0 & 0+j0 \\ 0+j0 & -0.00720793 + j0.01801984 & 0+j0 & 0+j0 & 0.19925780 - j0.49814451 & 0+j0 \\ 0+j0 & 0+j0 & -0.00720793 + j0.01801984 & 0+j0 & 0+j0 & 0.19925780 - j0.49814451 \end{bmatrix}$$

Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

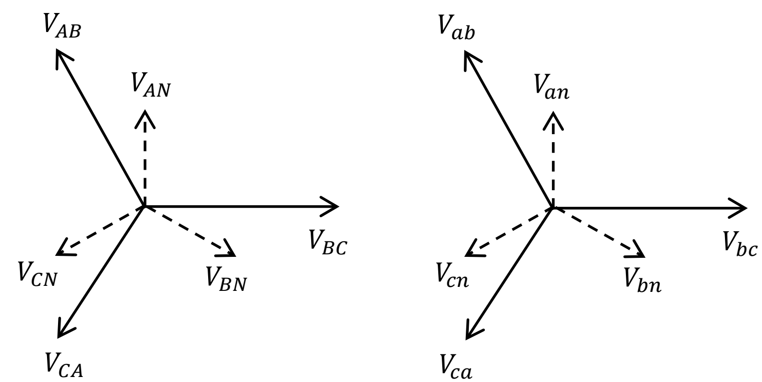
					G	B						
1	0.000261	-0.00065	0	0	0	0	-0.00721	0.01802	0	0	0	0
2	0	0	0.000261	-0.00065	0	0	0	0	-0.00721	0.01802	0	0
3	0	0	0	0	0.000261	-0.00065	0	0	0	0	-0.00721	0.01802
4	-0.00721	0.01802	0	0	0	0	0.199258	-0.49814	0	0	0	0
5	0	0	-0.00721	0.01802	0	0	0	0	0.199258	-0.49814	0	0
6	0	0	0	0	-0.00721	0.01802	0	0	0	0	0.199258	-0.49814
	1		2		3		4		5		6	

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



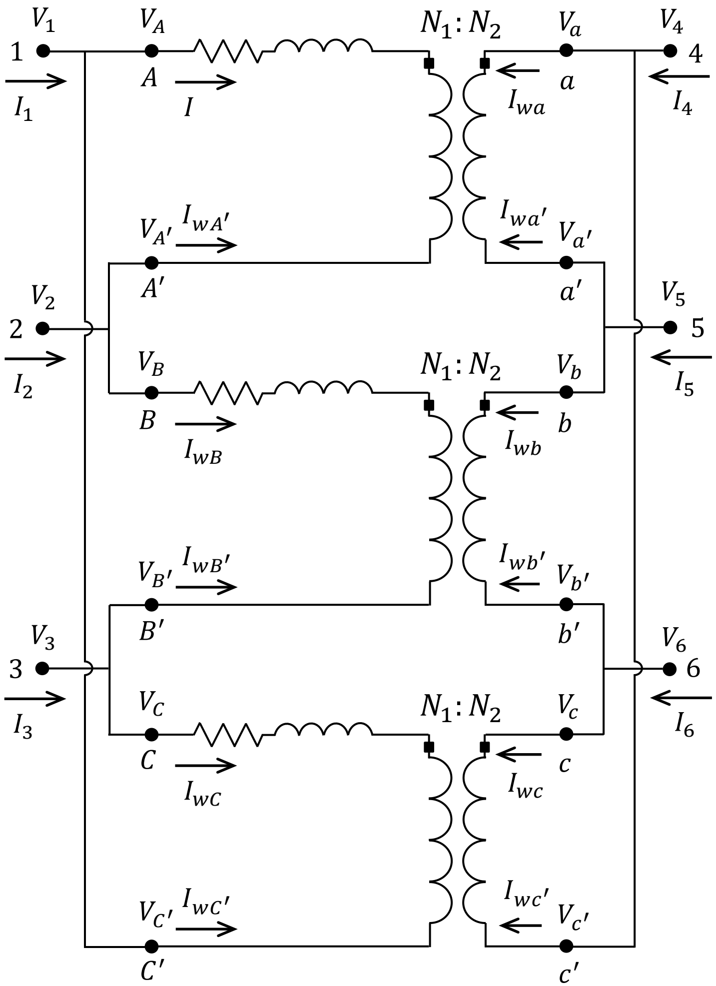
- The HV side always comes before the LV side, regardless of which is the primary winding.
- HV winding is taken as a reference.
- Phase rotation is always counterclockwise.
- 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

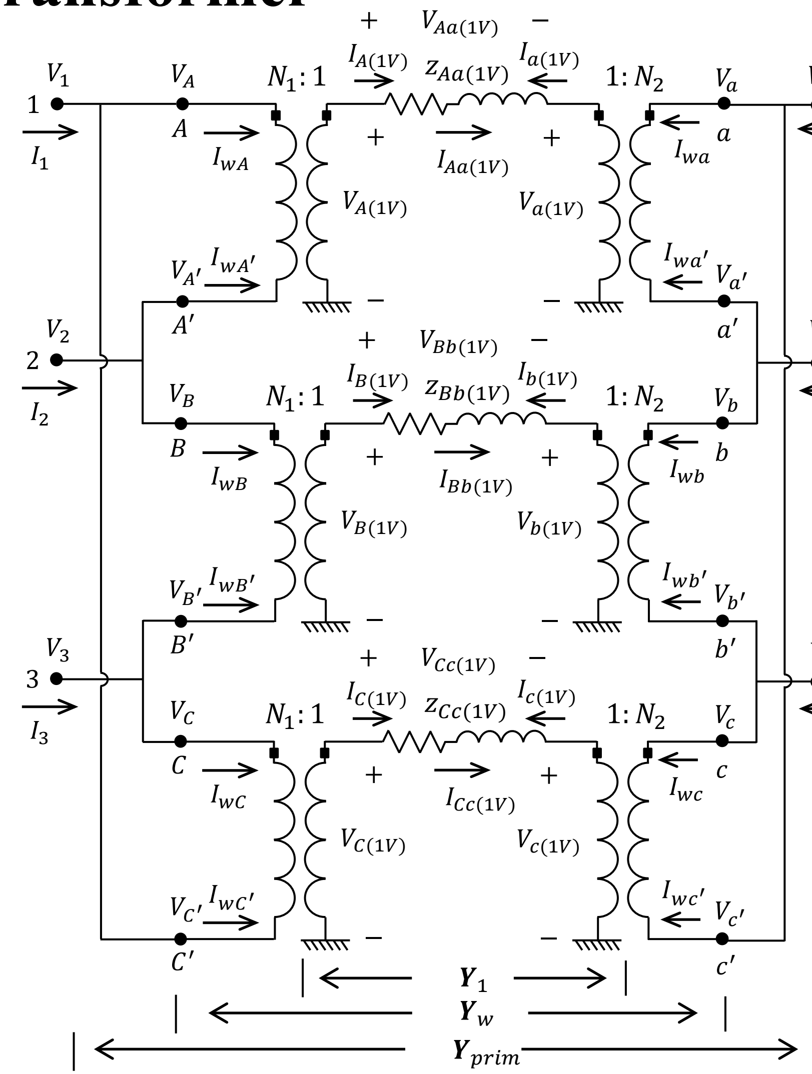


Dd0

Modeling A Transformer in OpenDSS

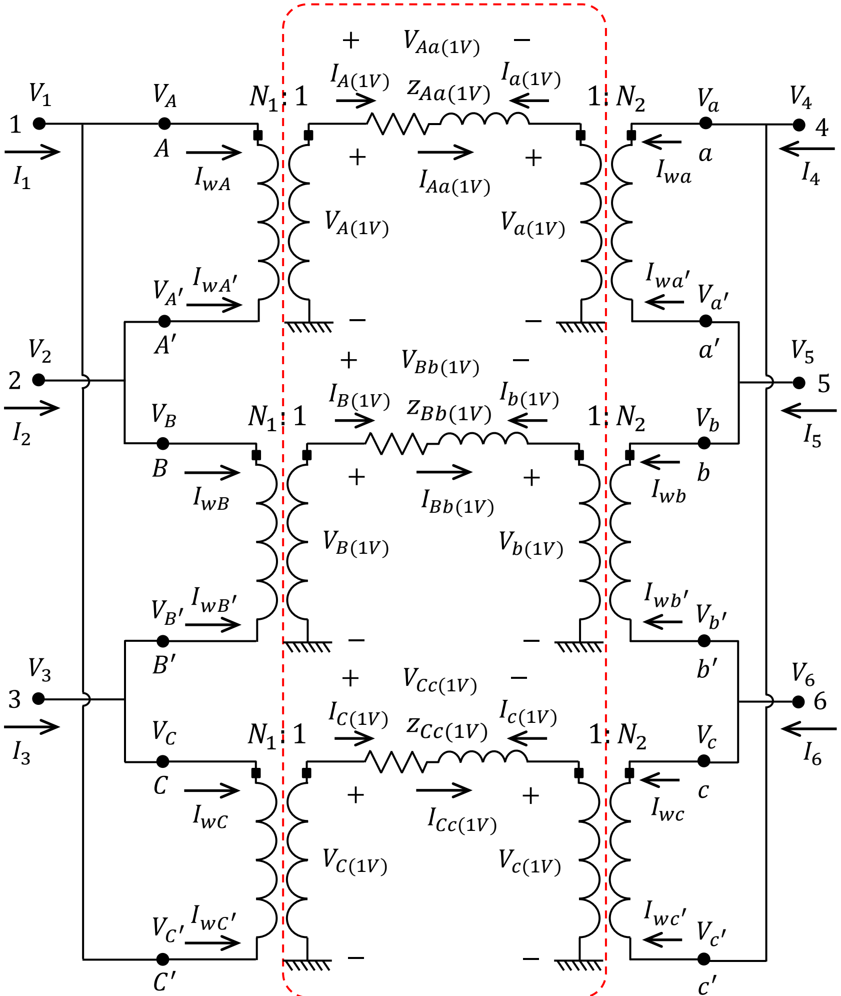
8. Three-phase Delta-Delta Transformer





Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Step 1: Compute the nodal admittance matrix on a one turn or one voltage base.

①
$$\begin{cases} I_{A(1V)} = I_{Aa(1V)} \\ I_{a(1V)} = -I_{Aa(1V)} \\ I_{B(1V)} = I_{Bb(1V)} \\ I_{b(1V)} = -I_{Bb(1V)} \\ I_{C(1V)} = I_{Cc(1V)} \\ I_{c(1V)} = -I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_1 = \boldsymbol{B}\boldsymbol{I}_{sc(1V)}, \text{ where, } \boldsymbol{I}_1 = \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \boldsymbol{I}_{sc(1V)} = \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

②
$$\begin{cases} V_{Aa(1V)} = V_{A(1V)} - V_{a(1V)} \\ V_{Bb(1V)} = V_{B(1V)} - V_{b(1V)} \\ V_{Cc(1V)} = V_{C(1V)} - V_{c(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix}$$

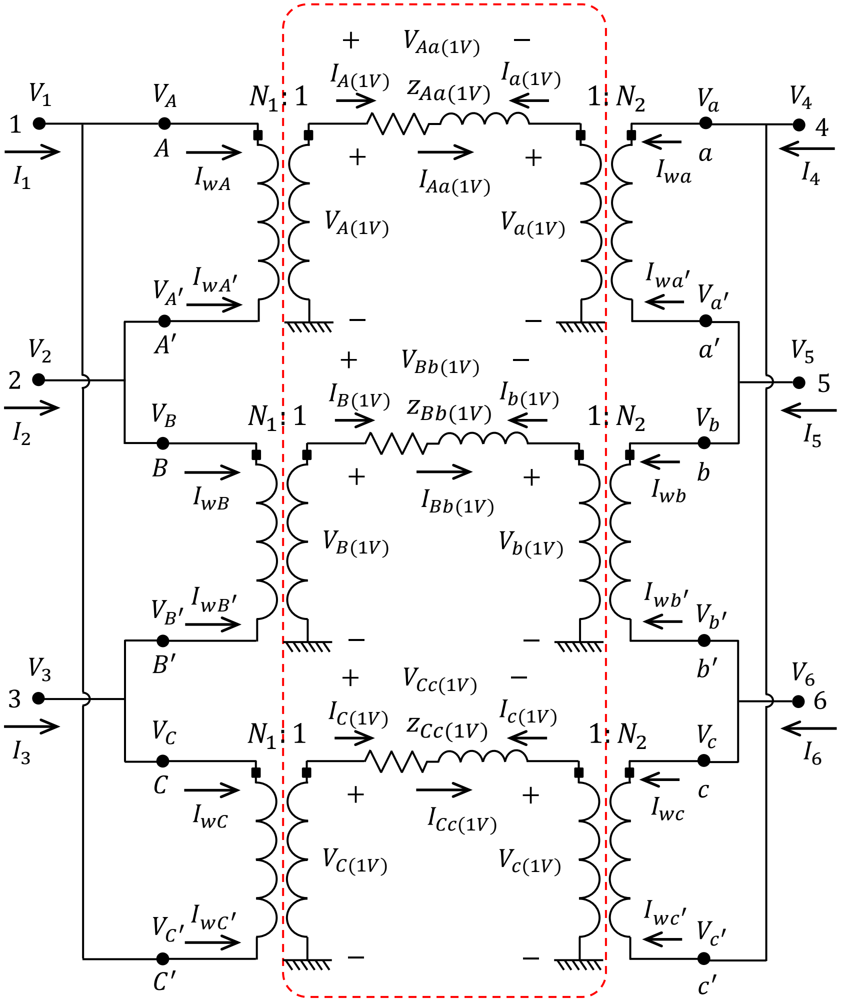
$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{B}^T \boldsymbol{V}_1, \text{ where, } \boldsymbol{V}_{sc(1V)} = \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix}, \boldsymbol{V}_1 = \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ \dot{V}_{c(1V)} \end{bmatrix}$$

③
$$\begin{cases} V_{Aa(1V)} = z_{Aa(1V)} I_{Aa(1V)} \\ V_{Bb(1V)} = z_{Bb(1V)} I_{Bb(1V)} \\ V_{Cc(1V)} = z_{Cc(1V)} I_{Cc(1V)} \end{cases} \Rightarrow \begin{bmatrix} V_{Aa(1V)} \\ V_{Bb(1V)} \\ V_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} z_{Aa(1V)} & 0 & 0 \\ 0 & z_{Bb(1V)} & 0 \\ 0 & 0 & z_{Cc(1V)} \end{bmatrix} \begin{bmatrix} I_{Aa(1V)} \\ I_{Bb(1V)} \\ I_{Cc(1V)} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_{sc(1V)} = \boldsymbol{Z}_{sc(1V)} \boldsymbol{I}_{sc(1V)} \Rightarrow \boldsymbol{Y}_{sc(1V)} \boldsymbol{V}_{sc(1V)} = \boldsymbol{I}_{sc(1V)}, \quad where, \boldsymbol{Y}_{sc(1V)} = (\boldsymbol{Z}_{sc(1V)})^{-1}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Let $\boldsymbol{I}_1 = \boldsymbol{Y}_1 \boldsymbol{V}_1$, where, $\boldsymbol{Y}_1$ is the admittance matrix on a one turn or one voltage base. What is $\boldsymbol{Y}_1$?

$$\left.\begin{aligned} \boldsymbol{I}_1 &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{B} \boldsymbol{I}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{B} \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \\ \boldsymbol{V}_{sc(1V)} &= \boldsymbol{B}^T \boldsymbol{V}_1 \end{aligned}\right\}$$

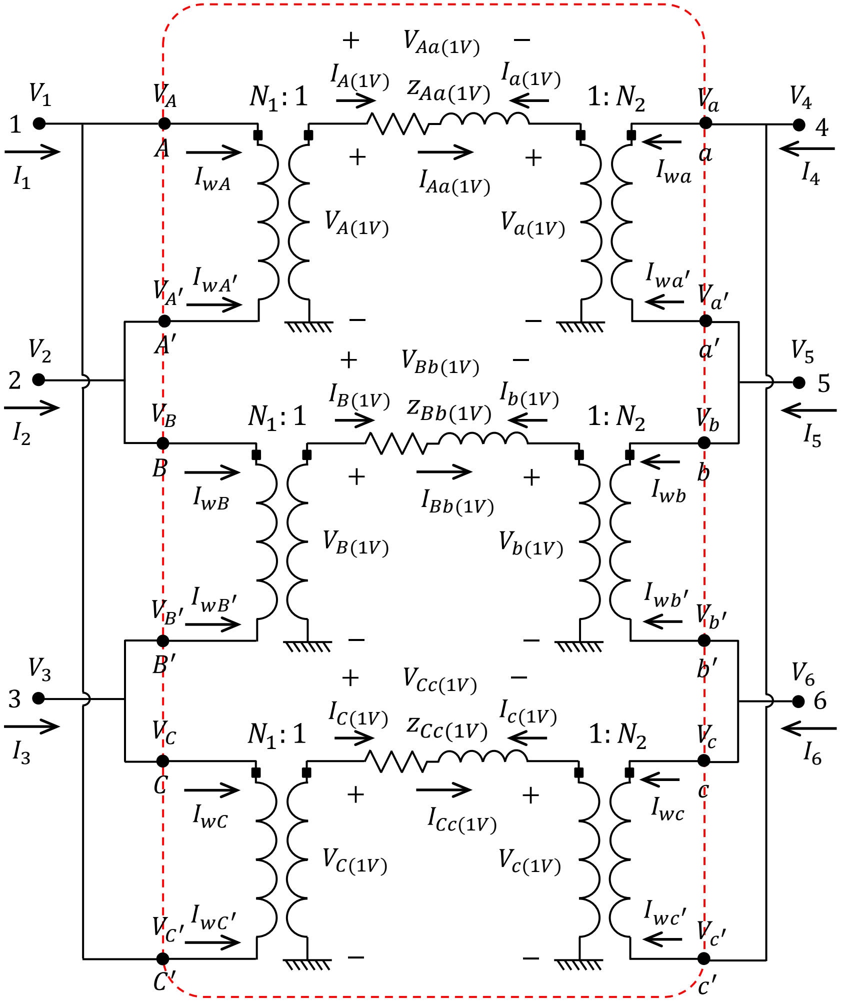
$$\Rightarrow \left.\begin{aligned} \boldsymbol{B} \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_1 \left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1} \boldsymbol{V}_{sc(1V)} \\ \boldsymbol{I}_{sc(1V)} &= \boldsymbol{Y}_{sc(1V)} \boldsymbol{V}_{sc(1V)} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{B} \boldsymbol{Y}_{sc(1V)} \boldsymbol{V}_{sc(1V)} = \boldsymbol{Y}_1 \left(\boldsymbol{B}^{\mathrm{T}}\right)^{-1} \boldsymbol{V}_{sc(1V)}$$

$$\Rightarrow \boldsymbol{Y}_1 = \boldsymbol{B} \boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Step 2: Consider winding turns ratio.

①
$$\begin{cases} I_{wA} = I_{A(1V)}/N_1 \\ I_{wA'} = -I_{A(1V)}/N_1 \\ I_{wa} = I_{a(1V)}/N_2 \\ I_{wa'} = -I_{a(1V)}/N_2 \\ I_{wB} = I_{B(1V)}/N_1 \\ I_{wB'} = -I_{B(1V)}/N_1 \\ I_{wb} = I_{b(1V)}/N_2 \\ I_{wb'} = -I_{b(1V)}/N_2 \\ I_{wC} = I_{C(1V)}/N_1 \\ I_{wC'} = -I_{C(1V)}/N_1 \\ I_{wc} = I_{c(1V)}/N_2 \\ I_{wc'} = -I_{c(1V)}/N_2 \end{cases}$$

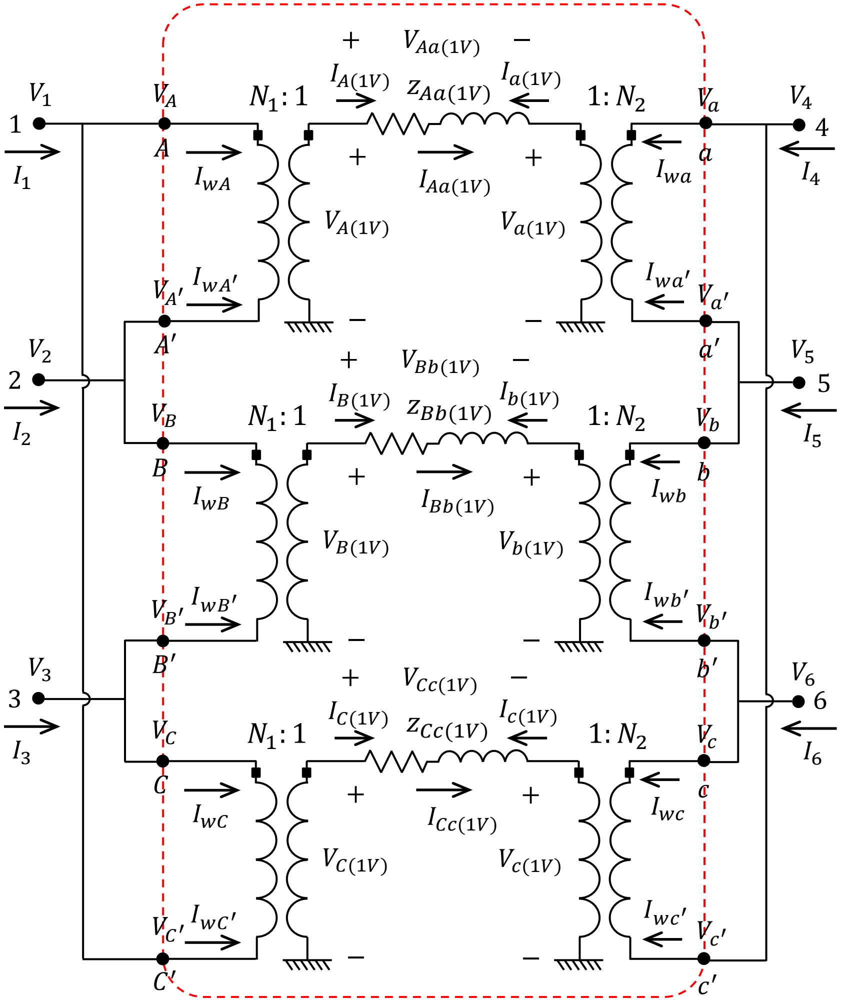
⇨

$$\begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} I_{A(1V)} \\ I_{a(1V)} \\ I_{B(1V)} \\ I_{b(1V)} \\ I_{C(1V)} \\ I_{c(1V)} \end{bmatrix}$$

⇨ $\boldsymbol{I}_w = \boldsymbol{N}\boldsymbol{I}_1$, where $\boldsymbol{I}_w = \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$, $\boldsymbol{N} = \begin{bmatrix} \frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{N_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{N_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{N_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{N_2} \end{bmatrix}$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



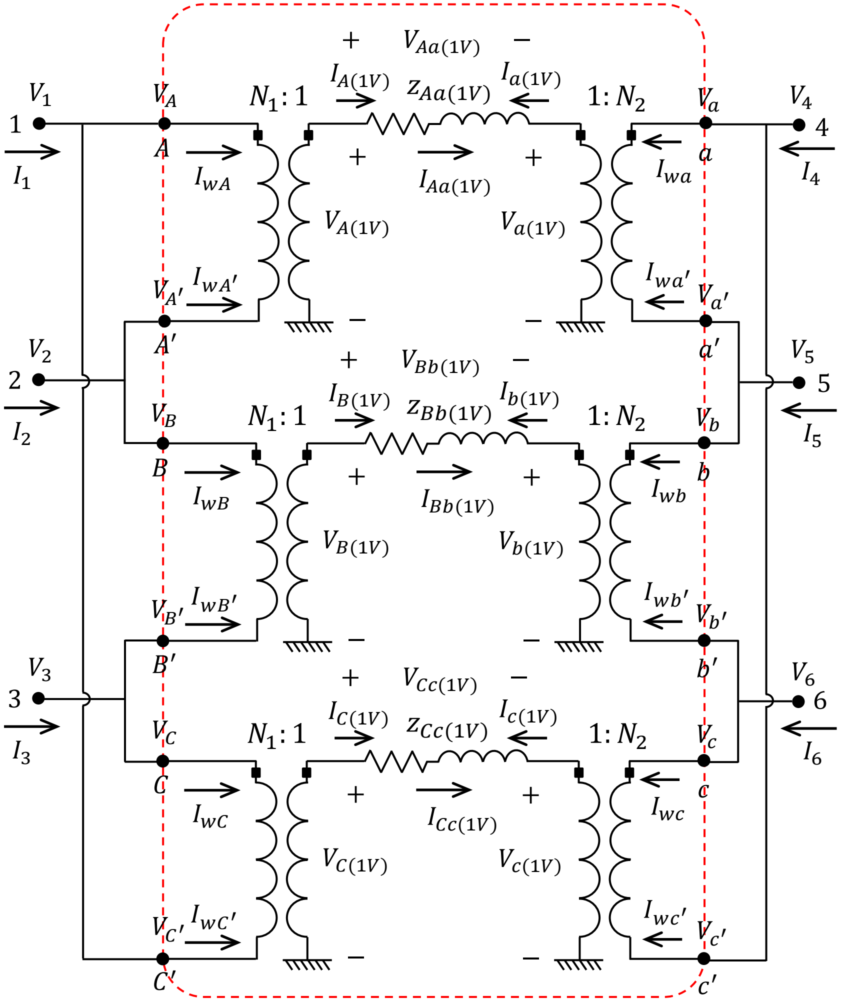
②
$$\begin{cases} V_{wA} - V_{wA'} = V_{A(1V)}N_1 \\ V_{wa} - V_{wa'} = V_{a(1V)}N_2 \\ V_{wB} - V_{wB'} = V_{B(1V)}N_1 \\ V_{wb} - V_{wb'} = V_{b(1V)}N_2 \\ V_{wC} - V_{wC'} = V_{C(1V)}N_1 \\ V_{wc} - V_{wc'} = V_{c(1V)}N_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} V_{A(1V)} \\ V_{a(1V)} \\ V_{B(1V)} \\ V_{b(1V)} \\ V_{C(1V)} \\ V_{c(1V)} \end{bmatrix} = \begin{bmatrix} \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_1} & -\frac{1}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{N_2} & -\frac{1}{N_2} \end{bmatrix} \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{V}_1 = \boldsymbol{N}^T \boldsymbol{V}_w, \text{ where, } \boldsymbol{v}_w = \begin{bmatrix} V_{wA} \\ V_{wA'} \\ V_{wa} \\ V_{wa'} \\ V_{wB} \\ V_{wB'} \\ V_{wb} \\ V_{wb'} \\ V_{wC} \\ V_{wC'} \\ V_{wc} \\ V_{wc'} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Next, let $\boldsymbol{I}_w = \boldsymbol{Y}_w \boldsymbol{V}_w$. How to obtain $\boldsymbol{Y}_w$?

$$\left.\begin{aligned} \boldsymbol{I}_w &= \boldsymbol{Y}_w \boldsymbol{V}_w \\ \boldsymbol{I}_w &= \boldsymbol{N}\boldsymbol{I}_1 \\ \boldsymbol{V}_1 &= \boldsymbol{N}^T \boldsymbol{V}_w \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{N}\boldsymbol{I}_1 &= \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1 \\ \boldsymbol{I}_1 &= \boldsymbol{Y}_1 \boldsymbol{V}_1 \end{aligned}\right\}$$

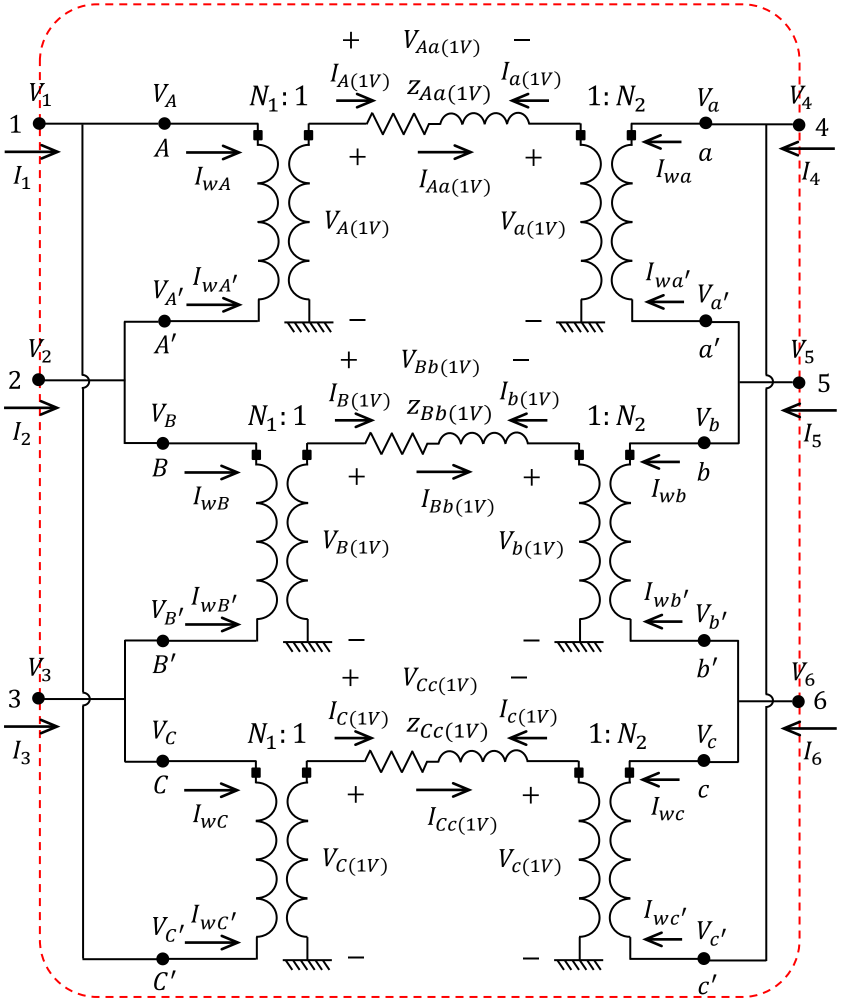
$$\Rightarrow \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{V}_1 = \boldsymbol{Y}_w (\boldsymbol{N}^T)^{-1} \boldsymbol{V}_1$$

$$\Rightarrow \left.\begin{aligned} \boldsymbol{Y}_w &= \boldsymbol{N}\boldsymbol{Y}_1\boldsymbol{N}^T \\ \boldsymbol{Y}_1 &= \boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}} \end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_w = \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)} \boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^T$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



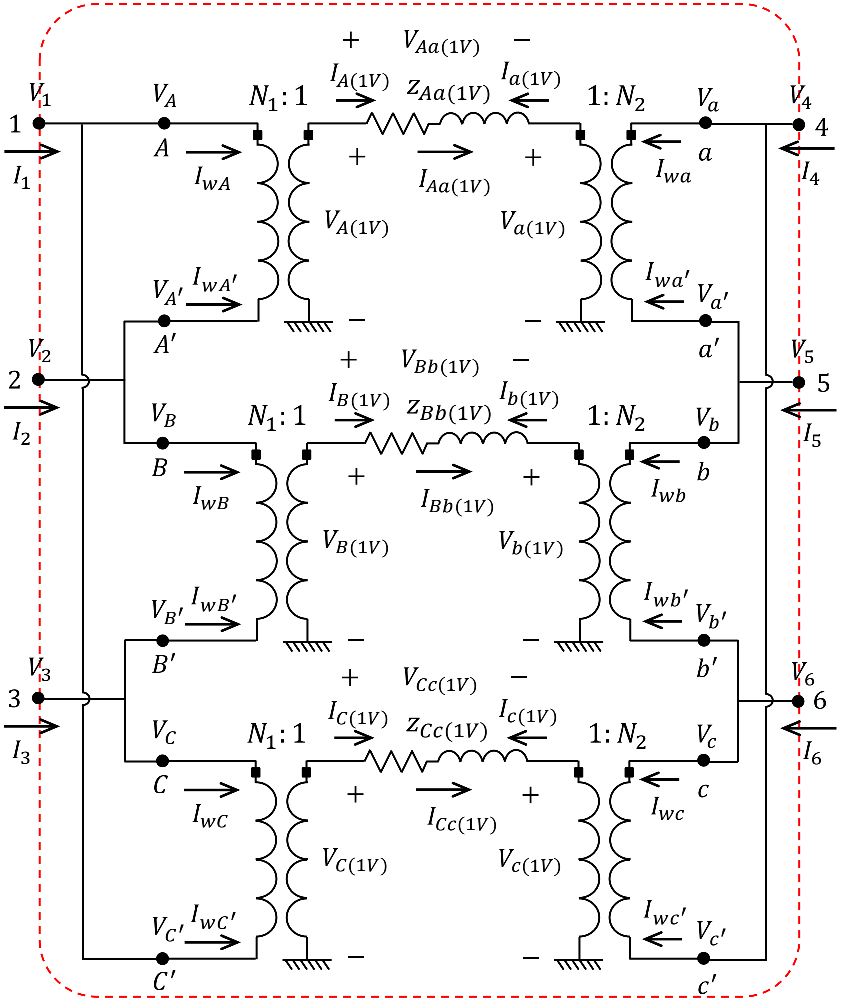
Step 3: Consider winding connections

①
$$\begin{cases} I_1 = I_{wA} + I_{wC'} \\ I_2 = I_{wB} + I_{wA'} \\ I_3 = I_{wC} + I_{wB'} \\ I_4 = I_{wa} + I_{wc'} \\ I_5 = I_{wb} + I_{wa'} \\ I_6 = I_{wc} + I_{wb'} \end{cases}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{wA} \\ I_{wA'} \\ I_{wa} \\ I_{wa'} \\ I_{wB} \\ I_{wB'} \\ I_{wb} \\ I_{wb'} \\ I_{wC} \\ I_{wC'} \\ I_{wc} \\ I_{wc'} \end{bmatrix}$$

$$\Rightarrow \boldsymbol{I}_{prim} = \boldsymbol{A}\boldsymbol{I}_w, \text{ where, } \boldsymbol{I}_{prim} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

8. Three-phase Delta-Delta Transformer



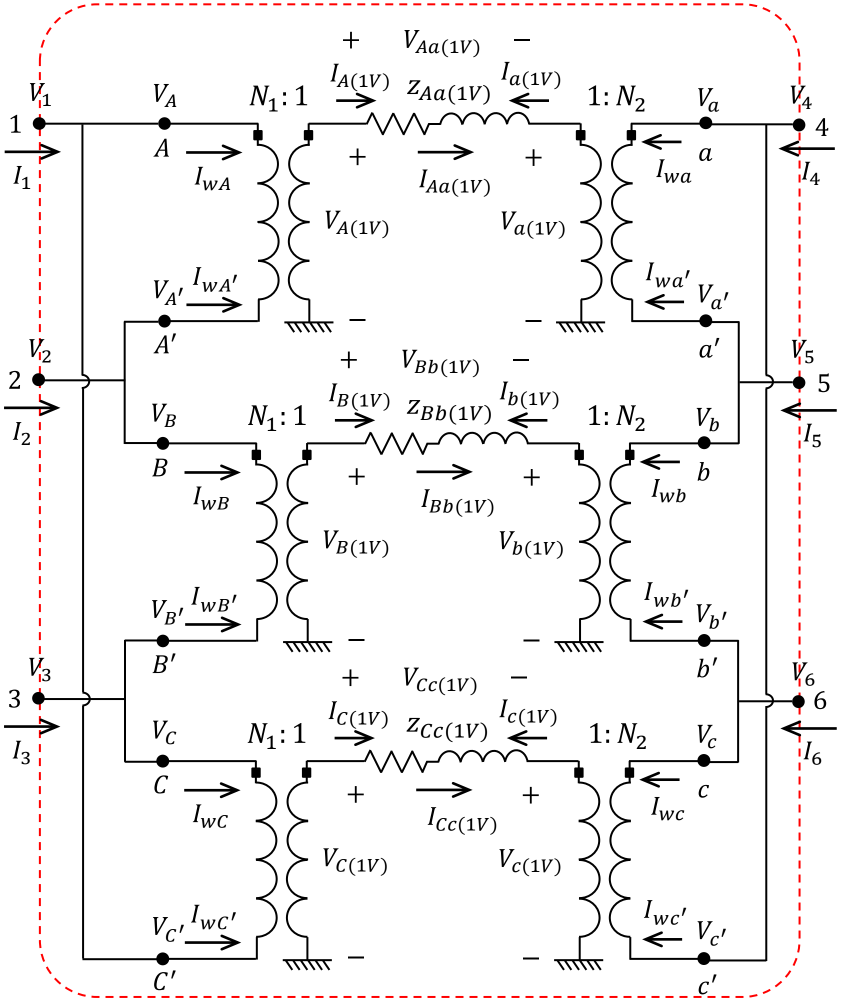
(2)

$$
\begin{cases}
V_A = V_1 \\
V_{A'} = V_2 \\
V_a = V_4 \\
V_{a'} = V_5 \\
V_B = V_2 \\
V_{B'} = V_3 \\
V_b = V_5 \\
V_{b'} = V_6 \\
V_C = V_3 \\
V_{C'} = V_1 \\
V_c = V_6 \\
V_{c'} = V_4
\end{cases}
\Rightarrow
\begin{bmatrix} V_A \\ V_{A'} \\ V_a \\ V_{a'} \\ V_B \\ V_{B'} \\ V_b \\ V_{b'} \\ V_C \\ V_{C'} \\ V_c \\ V_{c'} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}
$$

$$
\Rightarrow \boldsymbol{V}_w = \boldsymbol{A}^T \boldsymbol{V}_{prim}, \text{ where, } \boldsymbol{V}_{prim} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}.
$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer



Let $\boldsymbol{I}_{prim} = \boldsymbol{Y}_{prim}\boldsymbol{V}_{prim}$. How to obtain $\boldsymbol{Y}_{prim}$?

$$\left.\begin{aligned}\boldsymbol{I}_{prim} &= \boldsymbol{Y}_{prim}\boldsymbol{V}_{prim}\\ \boldsymbol{I}_{prim} &= \boldsymbol{A}\boldsymbol{I}_{w}\end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned}\boldsymbol{A}\boldsymbol{I}_{w} &= \boldsymbol{Y}_{prim}\boldsymbol{V}_{prim}\\ \boldsymbol{V}_{w} &= \boldsymbol{A}^{T}\boldsymbol{V}_{prim}\end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned}\boldsymbol{A}\boldsymbol{I}_{w} &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w}\\ \boldsymbol{I}_{w} &= \boldsymbol{Y}_{w}\boldsymbol{V}_{w}\end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned}\boldsymbol{A}\boldsymbol{Y}_{w}\boldsymbol{V}_{w} &= \boldsymbol{Y}_{prim}(\boldsymbol{A}^{T})^{-1}\boldsymbol{V}_{w}\\ \boldsymbol{Y}_{w} &= \boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\end{aligned}\right\}$$

$$\Rightarrow \boldsymbol{Y}_{prim} = \boldsymbol{A}\boldsymbol{N}\boldsymbol{B}\boldsymbol{Y}_{sc(1V)}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{N}^{T}\boldsymbol{A}^{T}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

An example:

```
// Define a three-phase transformer.
New Transformer.xfmr phases=3 windings=2 xhl=5
~ wdg=1 bus=K.1.2.3    conn=delta kV=115   kva=500 %r=1
~ wdg=2 bus=M.1.2.3    conn=delta kV=4.16  kva=500 %r=1
```

Transformer name
Number of phases
Number of windings
Percent reactance high-to-low
Specify which winding will be edited
Winding connection
Rated voltage
Base kVA rating
Percent resistance

Computing $\boldsymbol{Y}_{prim}$ using Matlab:

$$y_{Aa(1V)} = y_{Bb(1V)} = y_{Cc(1V)} = \frac{1}{[r(pu) + jx(pu)] * \frac{1^2}{S_t/3}} = \frac{500000}{3 * (0.02 + j0.05)} = 1149425.2873 \ - j2873563.2184$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\boldsymbol{N}^T = \begin{bmatrix} \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{115000} & -\frac{1}{115000} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4160} & -\frac{1}{4160} \end{bmatrix}$$

Modeling A Transformer in OpenDSS

8. Three-phase Delta-Delta Transformer

$$\boldsymbol{Y}_{sc(1V)} = \begin{bmatrix} y_{Aa(1V)} & 0 & 0 \\ 0 & y_{Bb(1V)} & 0 \\ 0 & 0 & y_{Cc(1V)} \end{bmatrix} = \begin{bmatrix} 1149425.2873 - j2873563.2184 & 0 & 0 \\ 0 & 1149425.2873 - j2873563.2184 & 0 \\ 0 & 0 & 1149425.2873 - j2873563.2184 \end{bmatrix}$$

Calculate $\boldsymbol{Y}_{prim} = \boldsymbol{ANBY}_{sc(1V)}\, \boldsymbol{B}^{\mathrm{T}} \boldsymbol{N}^{T}\, \boldsymbol{A}^{T} =$

$$\begin{bmatrix} 0.00017383 - j0.00043457 & -8.69131e-05 + j0.00021728 & -8.69131e-05 + j0.00021728 & -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 \\ -8.69131e-05 + j0.00021728 & 0.00017383 - j0.00043457 & -8.69131e-05 + j0.00021728 & 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 \\ -8.69131e-05 + j0.00021728 & -8.69131e-05 + j0.00021728 & 0.00017383 - j0.00043457 & 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 \\ -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 & 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 \\ 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 & 0.00240264 - j0.00600661 & -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 & -0.06641927 + j0.16604817 \\ 0.00240264 - j0.00600661 & 0.00240264 - j0.00600661 & -0.00480529 + j0.01201322 & -0.06641927 + j0.16604817 & -0.06641927 + j0.16604817 & 0.13283854 - j0.33209634 \end{bmatrix}$$

Exported $\boldsymbol{Y}_{prim}$ from OpenDSS:

					G	B						
1	0.000174	-0.00043	-8.69E-05	0.000217	-8.69E-05	0.000217	-0.00481	0.012013	0.002403	-0.00601	0.002403	-0.00601
2	-8.69E-05	0.000217	0.000174	-0.00043	-8.69E-05	0.000217	0.002403	-0.00601	-0.00481	0.012013	0.002403	-0.00601
3	-8.69E-05	0.000217	-8.69E-05	0.000217	0.000174	-0.00043	0.002403	-0.00601	0.002403	-0.00601	-0.00481	0.012013
4	-0.00481	0.012013	0.002403	-0.00601	0.002403	-0.00601	0.132839	-0.3321	-0.06642	0.166048	-0.06642	0.166048
5	0.002403	-0.00601	-0.00481	0.012013	0.002403	-0.00601	-0.06642	0.166048	0.132839	-0.3321	-0.06642	0.166048
6	0.002403	-0.00601	0.002403	-0.00601	-0.00481	0.012013	-0.06642	0.166048	-0.06642	0.166048	0.132839	-0.3321
	1		2		3		4		5		6	

Thank You!